Revision AT18 DP2

1a. *[2 marks]*

Each year the soccer team, Peterson United, plays 25 games at their home stadium. The owner of Peterson United claimed that last year the mean attendance per game at their home stadium was 24500.

Based on the owner's claim, calculate the total attendance for the games at Peterson United's home stadium last year.

1b. *[2 marks]*

The actual total attendance last year was 617700.

Calculate the percentage error in the owner's claim.

1c. *[2 marks]*

Write down your answer to **part (b)** in the form $a \times 10^k$ where $1 \leq a < 10$, $k \in \mathbb{Z}$.

2a. *[3 marks]*

Daniela is going for a holiday to South America. She flies from the US to Argentina stopping in Peru on the way.

In Peru she exchanges 85 United States dollars (USD) for Peruvian nuevo sol (PEN). The exchange rate is 1 USD = 3.25 PEN and a flat fee of 5 USD commission is charged.

Calculate the amount of PEN she receives.

2b. *[3 marks]*

At the end of Daniela's holiday she has 370 Argentinean peso (ARS). She converts this back to USD at a bank that charges a 4% commission on the exchange. The exchange rate is 1 USD = 9.60 ARS.

Calculate the amount of USD she receives.

3a. *[2 marks]*

The first three terms of a geometric sequence are $u_1 = 486,\ u_2 = 162,\ u_3 = 54$.

Find the value of r, the common ratio of the sequence.

3b. *[2 marks]*

Find the value of n for which $u_n = 2$.

3c. *[2 marks]*

Find the sum of the first 30 terms of the sequence.

4a. *[2 marks]*

Sergei is training to be a weightlifter. Each day he trains at the local gym by lifting a metal bar that has heavy weights attached. He carries out successive lifts. After each lift, the same amount of weight is **added** to the bar to increase the weight to be lifted.

The weights of each of Sergei's lifts form an arithmetic sequence.

Sergei's friend, Yuri, records the weight of each lift. Unfortunately, last Monday, Yuri misplaced all but two of the recordings of Sergei's lifts.

On that day, Sergei lifted 21 kg on the third lift and 46 kg on the eighth lift.

For that day find how much weight was added after each lift.

4b. *[2 marks]*

For that day find the weight of Sergei's first lift.

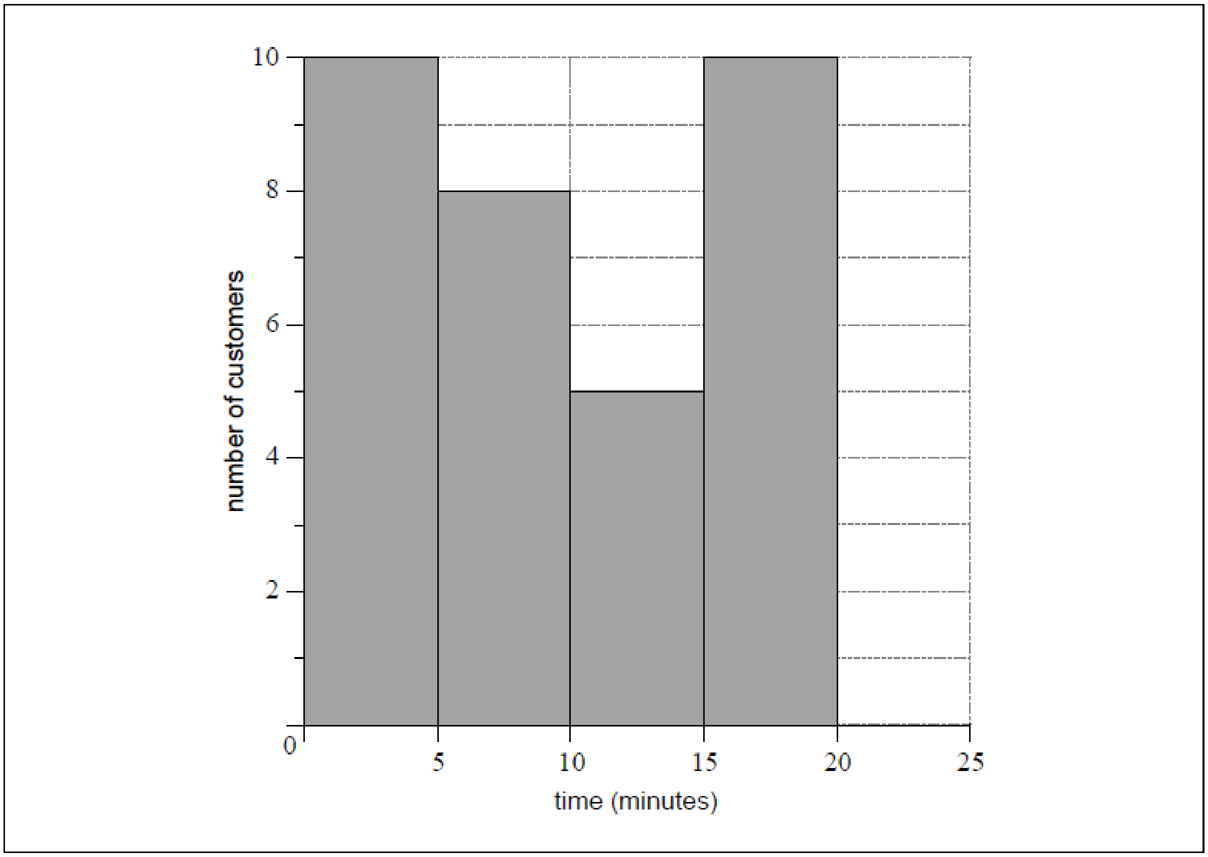
4c. *[2 marks]*

On that day, Sergei made 12 successive lifts. Find the total combined weight of these lifts.

5a. *[1 mark]*

The histogram shows the time, t, in minutes, that it takes the customers of a restaurant to eat their lunch on one particular day. Each customer took less than 25 minutes.

The histogram is incomplete, and only shows data for $0 \le t < 20$.



Write down the mid-interval value for $10 \le t < 15$.

5b. *[1 mark]*

The mean time it took **all** customers to eat their lunch was estimated to be 12 minutes.

It was found that k customers took between 20 and 25 minutes to eat their lunch.

Write down the total number of customers in terms of k.

5c. *[3 marks]*

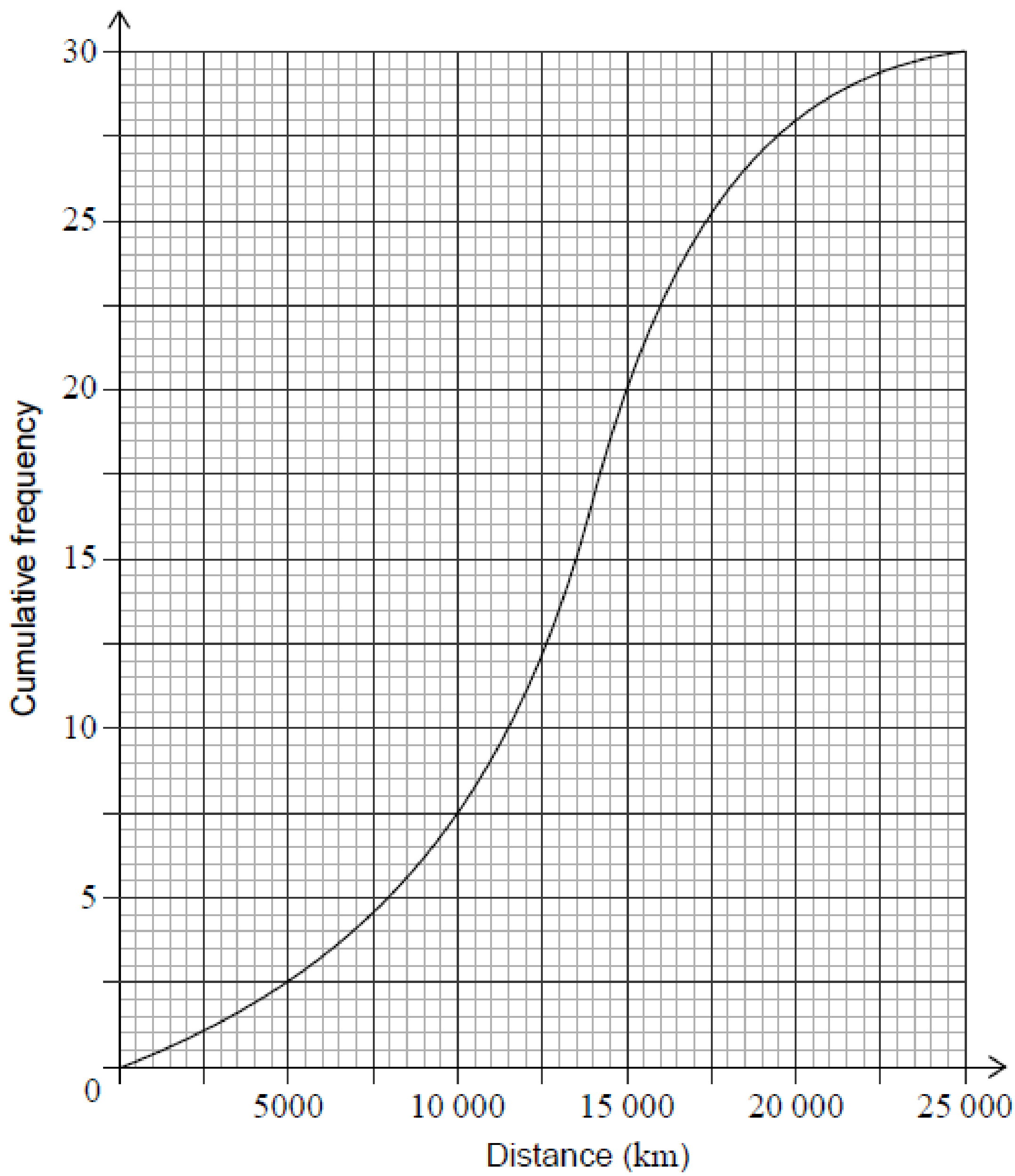
Calculate the value of k.

5d. *[1 mark]*

Hence, complete the histogram.

6a. *[2 marks]*

A transportation company owns 30 buses. The distance that each bus has travelled since being purchased by the company is recorded. The cumulative frequency curve for these data is shown.



Find the number of buses that travelled a distance between 15000 and 20000 kilometres.

6b. *[2 marks]*

Use the cumulative frequency curve to find the median distance.

6c. *[1 mark]*

Use the cumulative frequency curve to find the lower quartile.

6d. *[1 mark]*

Use the cumulative frequency curve to find the upper quartile.

6e. *[1 mark]*

Hence write down the interquartile range.

6f. *[1 mark]*

Write down the percentage of buses that travelled a distance greater than the upper quartile.

6g. *[1 mark]*

Find the number of buses that travelled a distance less than or equal to 12 000 km.

6h. *[2 marks]*

It is known that 8 buses travelled more than m kilometres.

Find the value of m.

6i. *[4 marks]*

The smallest distance travelled by one of the buses was 2500 km.
The longest distance travelled by one of the buses was 23 000 km.

On graph paper, draw a box-and-whisker diagram for these data. Use a scale of 2 cm to represent 5000 km.

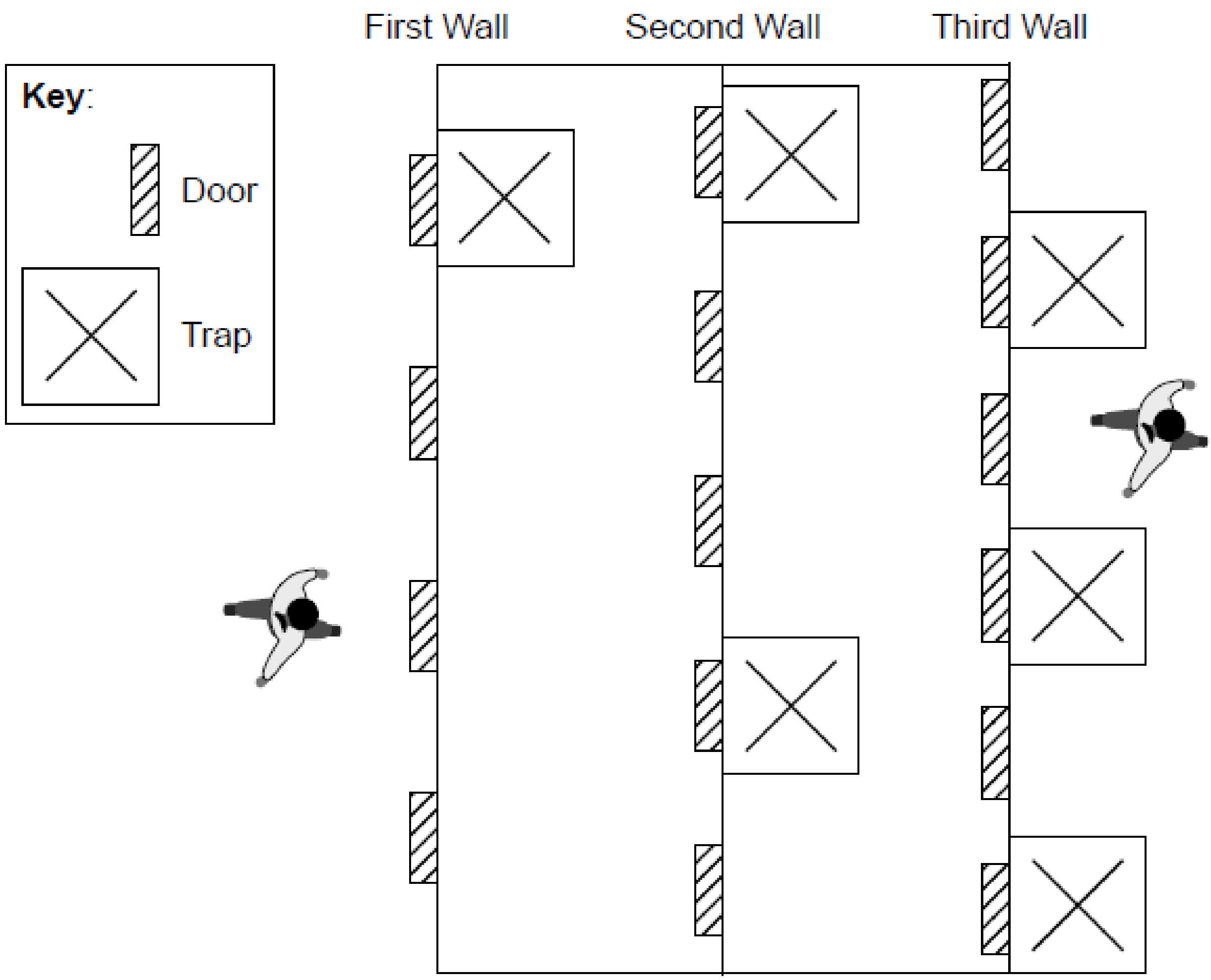
7a. *[1 mark]*

Contestants in a TV gameshow try to get through three walls by passing through doors without falling into a trap. Contestants choose doors at random.

If they avoid a trap they progress to the next wall.

If a contestant falls into a trap they exit the game before the next contestant plays.

Contestants are not allowed to watch each other attempt the game.



The first wall has four doors with a trap behind one door.

Ayako is a contestant.

Write down the probability that Ayako avoids the trap in this wall.

7b. *[3 marks]*

Natsuko is the second contestant.

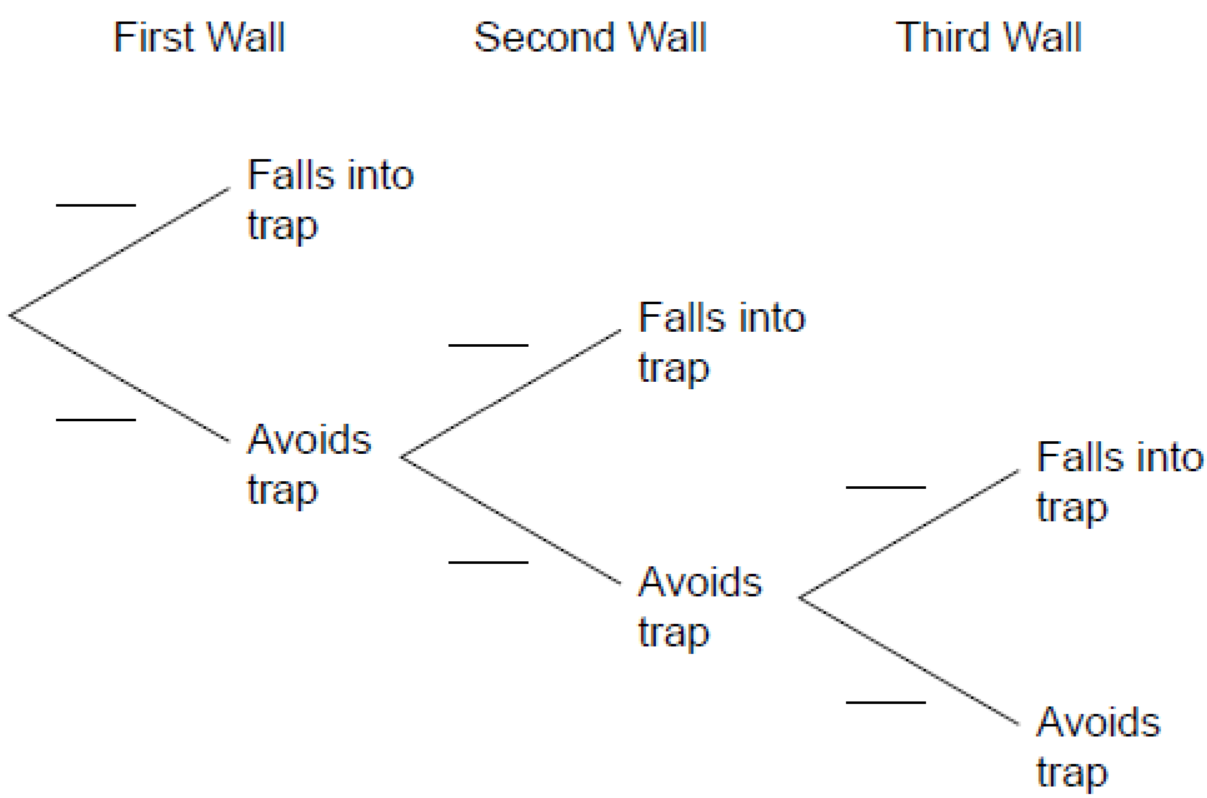
Find the probability that only one of Ayako and Natsuko falls into a trap while attempting to pass through a door **in the first wall**.

7c. *[3 marks]*

The second wall has five doors with a trap behind two of the doors.

The third wall has six doors with a trap behind three of the doors.

The following diagram shows the branches of a probability tree diagram for a contestant in the game.



Copy the probability tree diagram and write down the relevant probabilities along the branches.

7d. *[2 marks]*

A contestant is chosen at random. Find the probability that this contestant fell into a trap while attempting to pass through a door in the second wall.

7e. *[3 marks]*

A contestant is chosen at random. Find the probability that this contestant fell into a trap.

7f. *[3 marks]*

120 contestants attempted this game.

Find the expected number of contestants who fell into a trap while attempting to pass through a door in the third wall.

8a. *[3 marks]*

Consider the following propositions.

p: The car is under warranty
q: The car is less than 2 years old
r : The car has been driven more than 20 000 km

Write down in words $(q \vee \neg r) \Rightarrow p$.

8b. *[2 marks]*

Complete the truth table.

p	q	r	$\neg r$	$q \vee \neg r$	$(q \vee \neg r) \Rightarrow p$
T	T	T	F		
T	T	F	T		
T	F	T	F		
T	F	F	T		
F	T	T	F		
F	T	F	T		
F	F	T	F		
F	F	F	T		

8c. *[1 mark]*

State whether the statement $\neg p \Rightarrow \neg(q \vee \neg r)$ is the inverse, the converse or the contrapositive of the statement in part (a).

9a. *[1 mark]*

Consider the following propositions.

p: I completed the task
q: I was paid

Write down in words $\neg q$.

9b. *[1 mark]*

Write down in symbolic form the compound statement:

If I was paid then I completed the task.

9c. *[2 marks]*

Complete the following truth table.

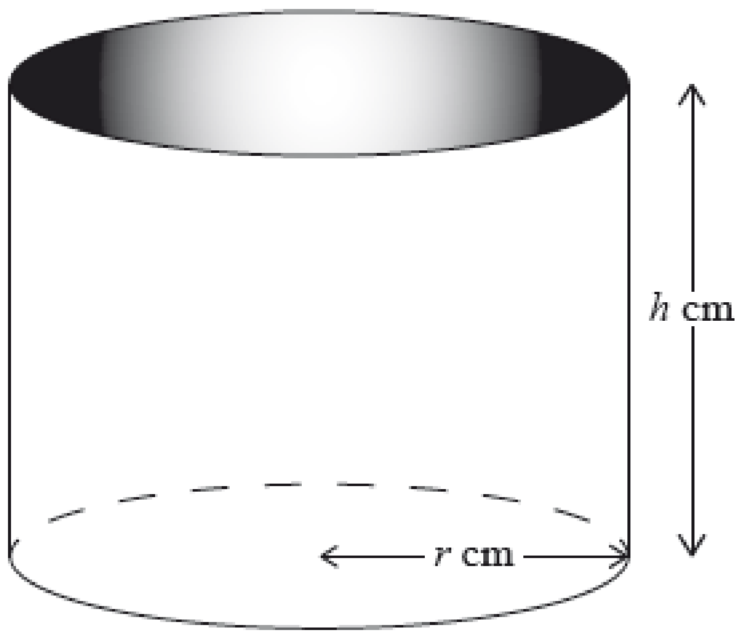
p	q	$\neg q$	$p \vee \neg q$	$q \Rightarrow p$
T	T	F		
T	F	T		
F	T	F		
F	F	T		

9d. *[2 marks]*

State whether the statements $p \vee \neg q$ and $q \Rightarrow p$ are logically equivalent. Give a reason for your answer.

10a. *[2 marks]*

A water container is made in the shape of a cylinder with internal height h cm and internal base radius r cm.



The water container has no top. The inner surfaces of the container are to be coated with a water-resistant material.

Write down a formula for A, the surface area to be coated.

10b. *[1 mark]*

The volume of the water container is $0.5\ \mathrm{m}^3$.

Express this volume in cm^3.

10c. *[1 mark]*

Write down, in terms of r and h, an equation for the volume of this water container.

10d. *[2 marks]*

Show that $A = \pi r^2 + \frac{1\,000\,000}{r}$.

10e. *[3 marks]*

The water container is designed so that the area to be coated is minimized.

Find $\frac{\mathrm{d}A}{\mathrm{d}r}$.

10f. *[3 marks]*

Using your answer to part (e), find the value of r which minimizes A.

10g. *[2 marks]*

Find the value of this minimum area.

10h. *[3 marks]*

One can of water-resistant material coats a surface area of $2000\ \mathrm{cm}^2$.

Find the least number of cans of water-resistant material that will coat the area in part (g).

11a. *[1 mark]*

An iron bar is heated. Its length, L, in millimetres can be modelled by a linear function, $L = mT + c$, where T is the temperature measured in degrees Celsius (°C).

At 150°C the length of the iron bar is 180 mm.

Write down an equation that shows this information.

11b. *[1 mark]*

At 210°C the length of the iron bar is 181.5 mm.

Write down an equation that shows this second piece of information.

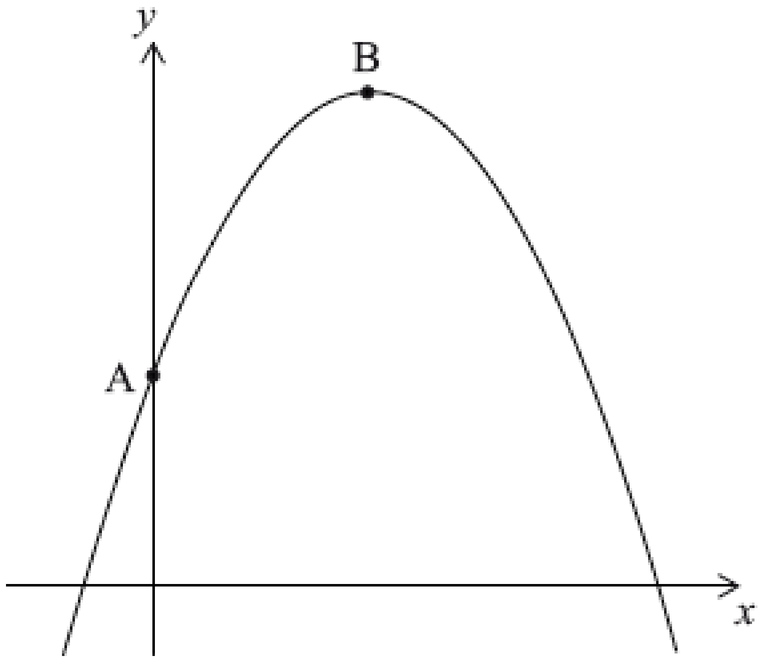
11c. *[4 marks]*

At 210°C the length of the iron bar is 181.5 mm.

Hence, find the length of the iron bar at 40°C.

12a. *[1 mark]*

The graph of the quadratic function $f(x) = ax^2 + bx + c$ intersects the *y*-axis at point A (0, 5) and has its vertex at point B (4, 13).



Write down the value of c.

12b. *[3 marks]*

By using the coordinates of the vertex, B, or otherwise, write down **two** equations in a and b.

12c. *[2 marks]*

Find the value of a and of b.

13a. *[1 mark]*

Consider the quadratic function $f\left(x\right) = ax^2 + bx + 22$.

The equation of the line of symmetry of the graph $y = f\left(x\right)$ is $x = 1.75$.

Using only this information, write down an equation in terms of *a* and *b*.

13b. *[1 mark]*

The graph intersects the *x*-axis at the point (−2 , 0).

Using this information, write down a second equation in terms of a and b.

13c. *[2 marks]*

Hence find the value of a and of b.

13d. *[2 marks]*

The graph intersects the x-axis at a second point, P.

Find the x-coordinate of P.
