

Paper 1

DP14 December 2016

58 min
70 marks

1. Let $m = 6.0 \times 10^3$ and $n = 2.4 \times 10^{-5}$.

Express each of the following in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(a) mn ;

(b) $\frac{m}{n}$.

Working: a) $6 \times 10^3 \times 2.4 \times 10^{-5}$ (calc: $6E3 \times 2.4E-5$)

b) (calc: $6E3 \div 2.4E-5$)

Answers:

(a) 1.44×10^{-1}

(b) 2.5×10^8

(Total 4 marks)

2. The exchange rate from US dollars (USD) to French francs (FFR) is given by $1 \text{ USD} = 7.5 \text{ FFR}$. Give the answers to the following correct to two decimal places.

- (a) Convert 115 US dollars to French francs.
- (b) Roger receives 600 Australian dollars (AUD) for 2430 FFR. Calculate the value of the US dollar in Australian dollars.

Working:

a) $1 \text{ USD} = 7.5 \text{ FFR}$
 $115 \text{ USD} = 7.5 \times 115 \text{ FFR}$

b) $600 \text{ AUD} = 2430 \text{ FFR}$
 $\frac{600}{2430} \text{ AUD} = 1 \text{ FFR} \quad (1 \text{ USD} = 7.5 \text{ FFR} \Rightarrow)$

$\frac{600}{2430} \times 7.5 \text{ AUD} = 7.5 \text{ FFR}$
 $(0.2469... \times 7.5 \text{ AUD} = 7.5 \text{ FFR})$

Answers:

(a) 862.50 USD

(b) 1.85 AUD

(Total 8 marks)

3. The first four terms of an arithmetic sequence are shown below.

1, 5, 9, 13,.....

- (a) Write down the n^{th} term of the sequence.
- (b) Calculate the 100th term of the sequence.
- (c) Find the sum of the first 100 terms of the sequence.

<p><i>Working:</i></p> <p>a) $u_n = u_1 + (n-1)d$ $u_n = 1 + (n-1)4$ $u_n = 1 + 4n - 4$ $u_n = 4n - 3$</p> <p>b) $(n=100 \Rightarrow)$ $4 \times 100 - 3 = 397$</p>	<p>c) $(S_n = \frac{n}{2} (u_1 + u_n))$ $S_{100} = \frac{100}{2} (1 + 397)$ $S_{100} = 50 \times 398$ $(S_{100} = 19900)$</p>
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Answers:

(a) $u_n = 4n - 3$

(b) 397

(c) 19900

(Total 4 marks)

4. At what interest rate, compounded annually, would you need to invest \$100 in order to have \$125 in 2 years?

Working:

$$100 \times x^2 = 125$$

$$x^2 = 1.25$$

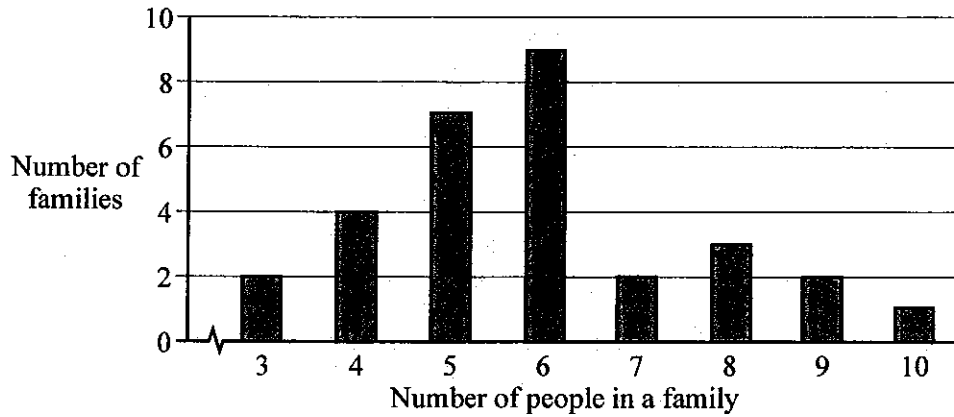
$$x = 1.118$$

Answer:

..... 11.8%

(Total 4 marks)

5. The bar chart below shows the number of people in a selection of families.



- (a) How many families are represented?
- (b) Write down the mode of the distribution.
- (c) Find, correct to the nearest whole number, the mean number of people in a family.

Working:

$$a) (2 + 4 + 7 + 9 + 2 + 3 + 2 + 1)$$

$$b) \left(\frac{30+1}{2} \Rightarrow 15^{\text{th}} \text{ and } 16^{\text{th}} \text{ observation} \right)$$

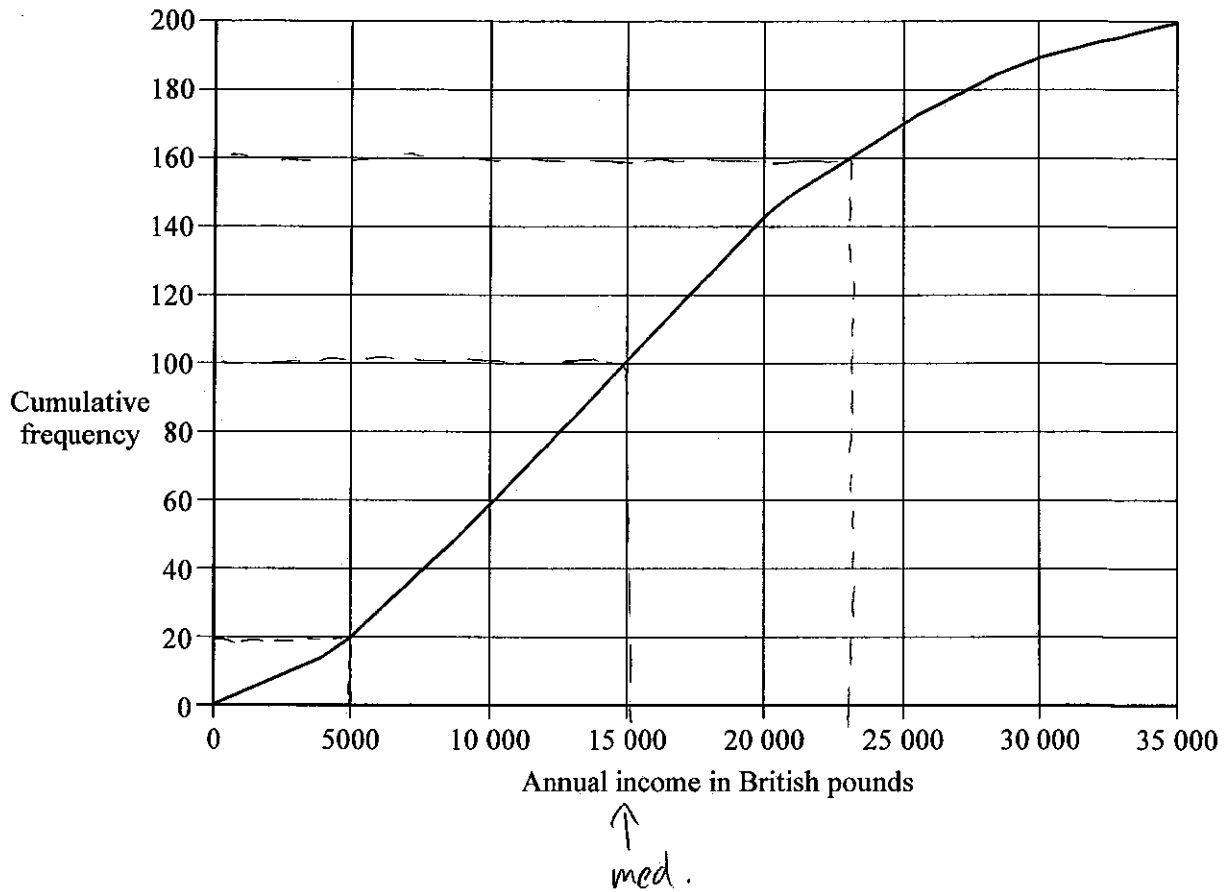
$$c) \left(\frac{3 \times 2 + 4 \times 4 + 5 \times 7 + 6 \times 9 + 2 \times 7 + 3 \times 8 + 2 \times 9 + 1 \times 10}{30} \right) = 5.9$$

Answers:

- (a) 30
- (b) 6
- (c) 6

(Total 4 marks)

6. The graph below shows the cumulative frequency for the yearly incomes of 200 people.



Use the graph to estimate

- (a) the number of people who earn less than 5000 British pounds per year;
- (b) the median salary of the group of 200 people;
- (c) the lowest income of the richest 20% of this group.

Working:

a)

b)

c) 80% of 200 (0.8×200) = 160

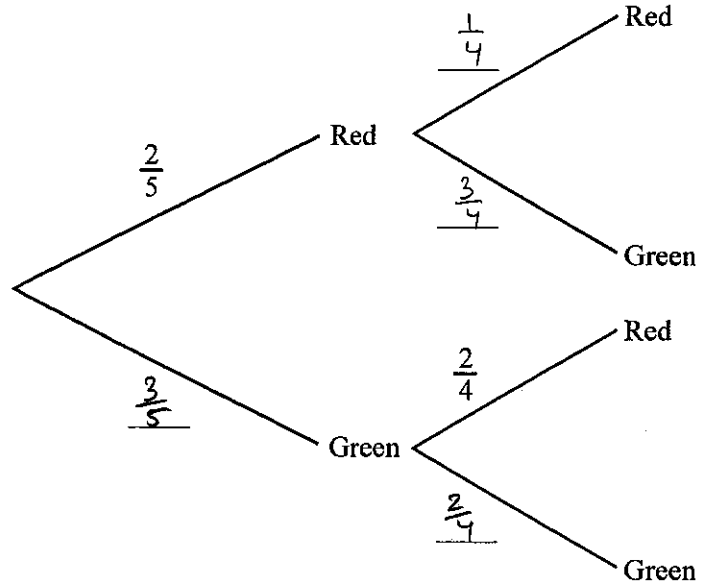
Answers:

- (a) 20
- (b) 15000 GBP
- (c) 23000 GBP

(Total 4 marks)

7. A bag contains two red sweets and three green sweets. Jacques takes one sweet from the bag, notes its colour, then eats it. He then takes another sweet from the bag.

Complete the tree diagram below to show all probabilities.



Working:

(Total 4 marks)

8. Consider the following logic statements:

p : the train arrives on time
 q : I am late for school

(a) Write the expression $p \Rightarrow \neg q$ as a logic statement.

(b) Write the following statement in logic symbols:

"The train does not arrive on time and I am not late for school."

(c) Complete the following truth table.

p	q	$\neg p$	$\neg q$	$p \Rightarrow \neg q$	$\neg p \wedge \neg q$
T	T	F	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	F	T	T	T	T

(d) Are the two compound propositions $(p \Rightarrow \neg q)$ and $(\neg p \wedge \neg q)$ logically equivalent?

Working:

a)

Answers:

- (a) If the train arrives on time then I am not late for school
- (b) $\neg p \wedge \neg q$
- (d) No

(Total 8 marks)

9. The universal set U is defined as the set of positive integers less than 10. The subsets A and B are defined as:

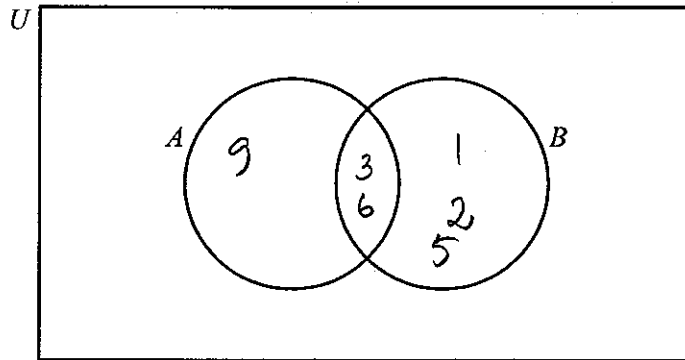
$$A = \{\text{integers that are multiples of 3}\}$$

$$B = \{\text{integers that are factors of 30}\}$$

(a) List the elements of

- (i) A ;
- (ii) B .

(b) Place the elements of A and B in the appropriate region in the Venn diagram below.



Working:

$$a) \text{ i) } A = \{3, 6, 9\}$$

$$\text{ii) } B = \{1, 2, 3, 5, 6\}$$

Answers:

- (a) (i) $A = \{3, 6, 9\}$
 (ii) $B = \{1, 2, 3, 5, 6\}$

(Total 4 marks)

10. At Jumbo's Burger Bar, Jumbo burgers cost £ J each and regular cokes cost £ C each. Two Jumbo burgers and three regular cokes cost £5.95.

(a) Write an equation to show this.

(b) If one Jumbo Burger costs £2.15, what is the cost, in pence, of one regular coke?

Working:

$$2J + 3C = 5.95$$

$$2 \times 2.15 + 3C = 5.95$$

$$3C = 5.95 - 4.3$$

$$3C = 1.65$$

$$C = 0.55$$

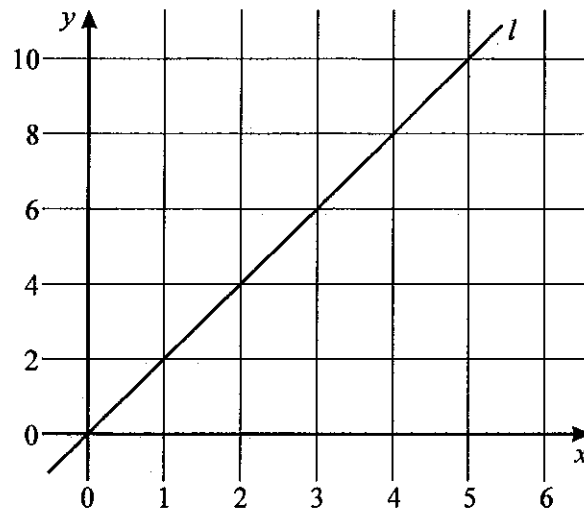
Answers:

(a) $2J + 3C = 5.95$

(b) £0.55

(Total 4 marks)

11. The following diagram shows a straight line l .



- (a) Find the equation of the line l .
- (b) The line n is parallel to l and passes through the point $(0, 8)$. Write down the equation of the line n .
- (c) The line n crosses the horizontal axis at the point P. Find the coordinates of P.

Working:

$$a) (0,0) (1,2) \quad m = \frac{2-0}{1-0} = 2$$

b)

$$c) y = 2x + 8 \quad y = 0 \Rightarrow$$

$$0 = 2x + 8$$

$$-8 = 2x$$

$$x = -4$$

Answers:

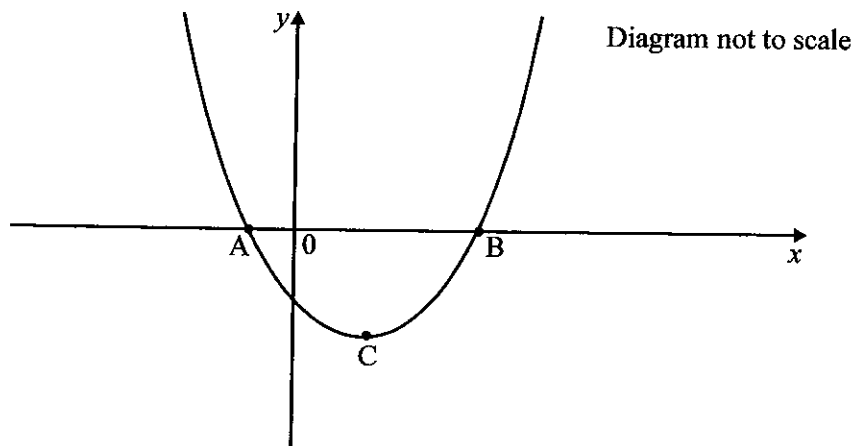
(a) $y = 2x$

(b) $y = 2x + 8$

(c) $(-4, 0)$

(Total 4 marks)

12. The graph of the function $f(x) = x^2 - 2x - 3$ is shown in the diagram below.



- (a) Factorize the expression $x^2 - 2x - 3$.
- (b) Write down the coordinates of the points A and B.
- (c) Write down the equation of the axis of symmetry.
- (d) Write down the coordinates of the point C, the vertex of the parabola.

Working:

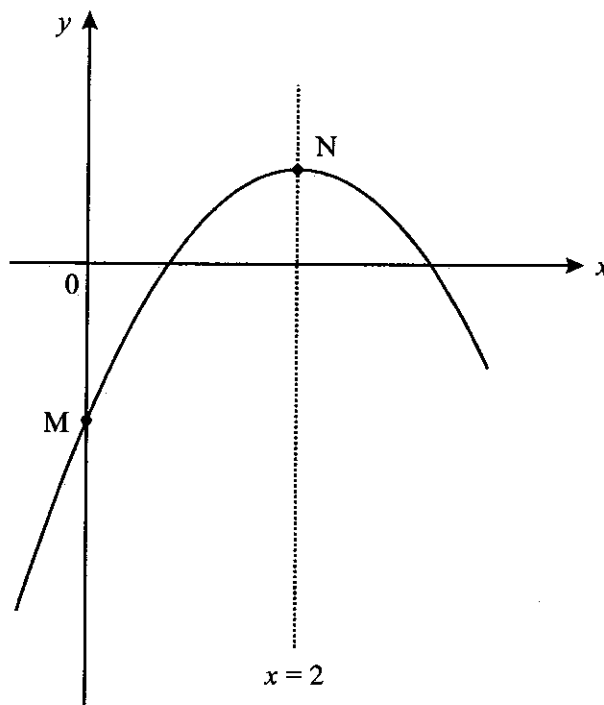
$$d) x = 1 \Rightarrow y = 1^2 - 2 \times 1 - 3 = -4$$

Answers:

- (a) $(x + 1)(x - 3)$
- (b) $(-1, 0), (3, 0)$
- (c) $x = 1$
- (d) $(1, -4)$

(Total 8 marks)

13. The diagram below shows part of the graph of $y = ax^2 + 4x - 3$. The line $x = 2$ is the axis of symmetry. M and N are points on the curve, as shown.



- (a) Find the value of a .
- (b) Find the coordinates of
- M;
 - N.

Working:

a) $\left(x = \frac{-b}{2a}\right)$ formula for axis of symmetry

$$2 = \frac{-4}{2a}$$

$$4a = -4$$

$$a = -1$$

c) $x = 2 \Rightarrow y = -1 \times 2^2 + 4 \times 2 - 3$
 $y = -4 + 8 - 3 = 1$

Answers:

- (a) -1
- (b) (i) (0, -3)
- (ii) (2, 1)

(Total 4 marks)

14. A function is represented by the equation

$$f(x) = ax^2 + \frac{4}{x} - 3.$$

(a) Find $f'(x)$.

(3)

The function $f(x)$ has a local maximum at the point where $x = -1$.

(b) Find the value of a .

(3)

Working:

$$a) f(x) = ax^2 + 4x^{-1} - 3$$

$$f'(x) = 2ax - 4x^{-2} = 2ax - \frac{4}{x^2}$$

$$b) (\text{where } x = -1 \text{ } f'(x) = 0)$$

$$2a \times (-1) - \frac{4}{(-1)^2} = 0$$

$$-2a - 4 = 0$$

$$-4 = 2a$$

$$a = -2$$

Answers:

(a) $2ax - 4x^{-2}$

(b) $a = -2$

(Total 6 marks)