

Geometry 2 DP2

1a. [4 marks]

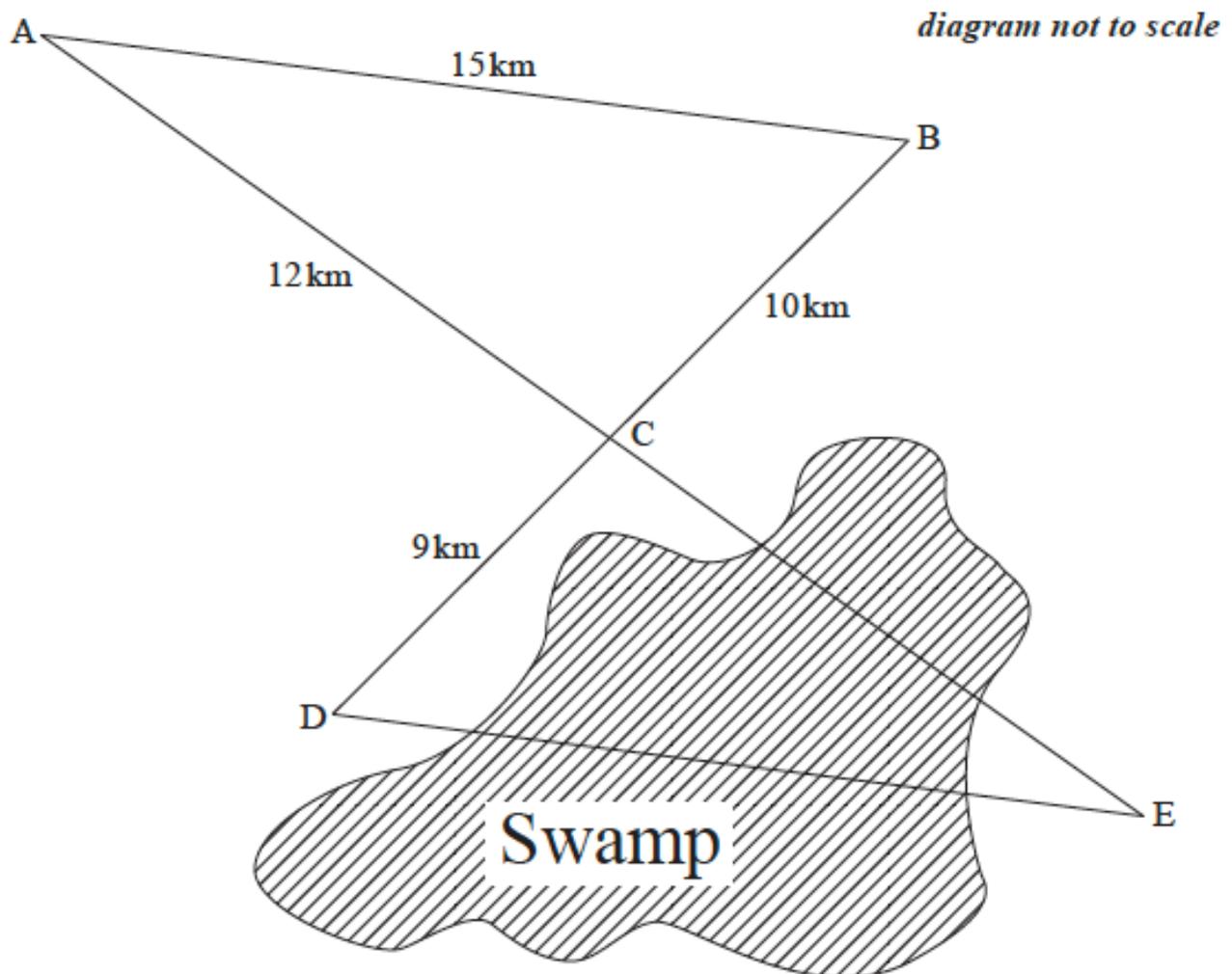
A surveyor has to calculate the area of a triangular piece of land, DCE.

The lengths of CE and DE cannot be directly measured because they go through a swamp.

AB, DE, BD and AE are straight paths. Paths AE and DB intersect at point C.

The length of AB is 15 km, BC is 10 km, AC is 12 km, and DC is 9 km.

The following diagram shows the surveyor's information.



(i) Find the size of angle ACB.

(ii) Show that the size of angle DCE is 85.5° , correct to one decimal place.

1b. [5 marks]

The surveyor measures the size of angle **CDE** to be twice that of angle **DEC**.

(i) Using angle **DCE** = 85.5° , find the size of angle **DEC**.

(ii) Find the length of **DE**.

1c. [4 marks]

Calculate the area of triangle **DEC**.

2a. [3 marks]

A cuboid has a rectangular base of width x cm and length $2x$ cm . The height of the cuboid is h cm . The total length of the edges of the cuboid is **72** cm.

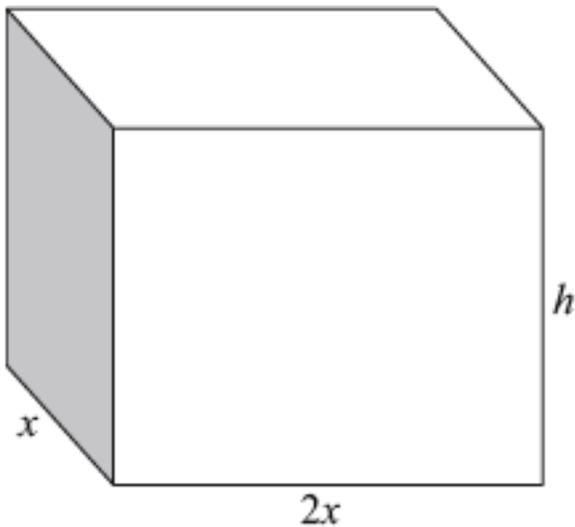


diagram not to scale

The volume, V , of the cuboid can be expressed as $V = ax^2 - 6x^3$.

Find the value of a .

2b. [3 marks]

Find the value of x that makes the volume a maximum.

3a. [2 marks]

Fabián stands on top of a building, T, which is on a horizontal street.

He observes a car, C, on the street, at an angle of depression of 30° . The base of the building is at B. The height of the building is 80 metres.

The following diagram indicates the positions of T, B and C.

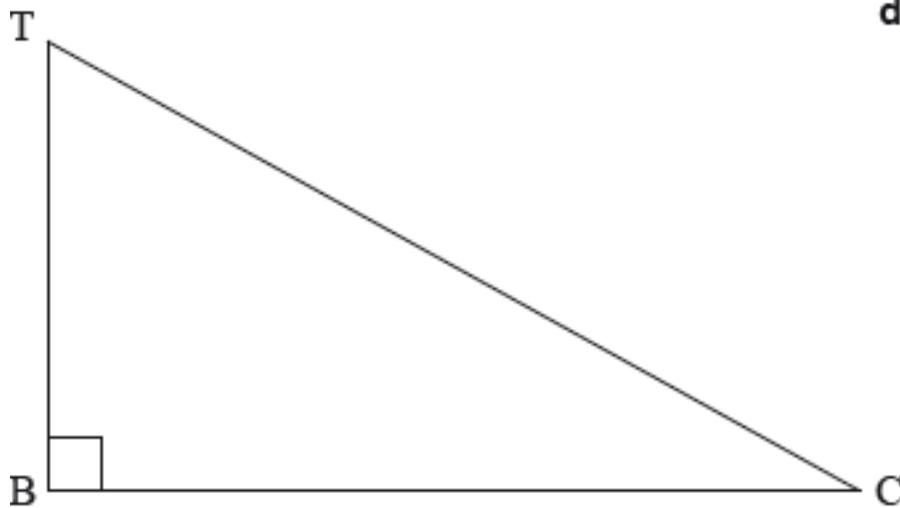


diagram not to scale

Show, in the appropriate place on the diagram, **the values** of

- (i) the height of the building;
- (ii) the angle of depression.

3b. [2 marks]

Find the distance, BC, from the base of the building to the car.

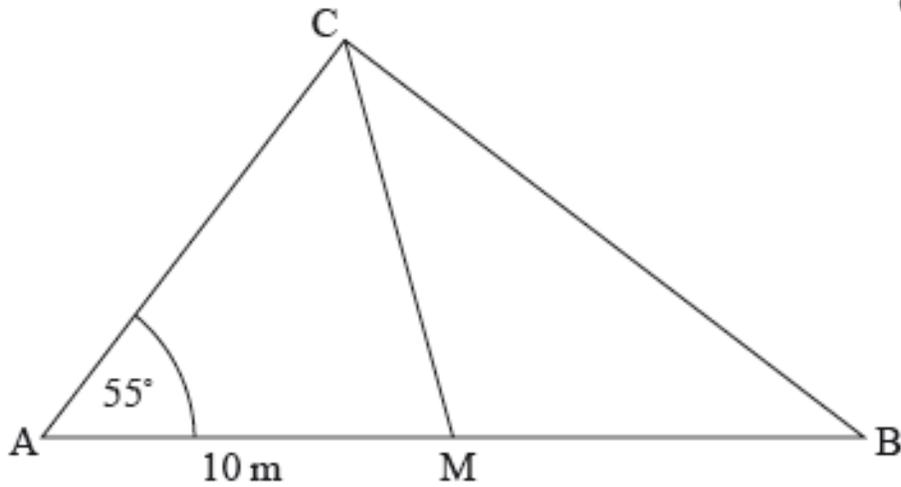
3c. [2 marks]

Fabián estimates that the distance from the base of the building to the car is 150 metres. Calculate the percentage error of Fabián's estimate.

4a. [1 mark]

The diagram shows a triangle **ABC**. The size of angle **CÂB** is **55°** and the length of **AM** is **10 m**, where **M** is the midpoint of **AB**. Triangle **CMB** is isosceles with **CM = MB**.

diagram not to scale



Write down the length of MB.

4b. [2 marks]

Find the size of angle \widehat{CMB} .

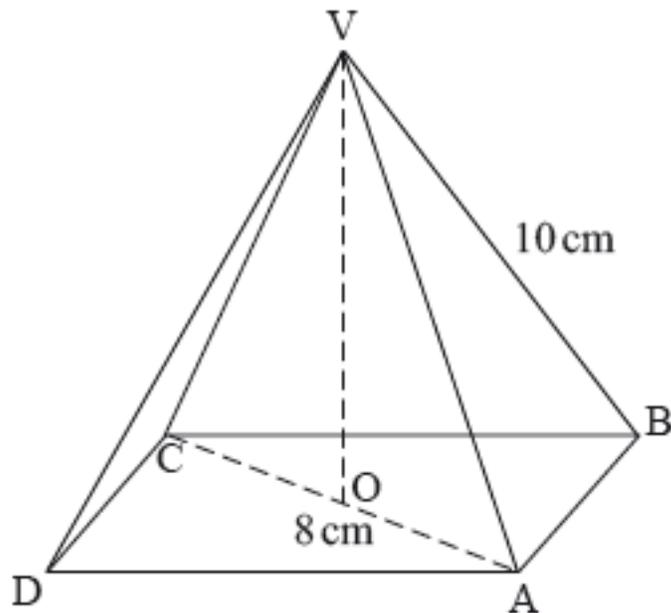
4c. [3 marks]

Find the length of CB.

5a. [1 mark]

In the following diagram, ABCD is the square base of a right pyramid with vertex V. The centre of the base is O. The diagonal of the base, AC, is 8 cm long. The sloping edges are 10 cm long.

diagram not to scale



Write down the length of **AO**.

5b. [2 marks]

Find the size of the angle that the sloping edge **VA** makes with the base of the pyramid.

5c. [3 marks]

Hence, or otherwise, find the area of the triangle **CAV**.

6a. [2 marks]

A cuboid has the following dimensions: length = 8.7 cm, width = 5.6 cm and height = 3.4 cm.

Calculate the **exact** value of the volume of the cuboid, in cm.

6b. [2 marks]

Write your answer to part (a) correct to

(i) one decimal place;

(ii) three significant figures.

6c. [2 marks]

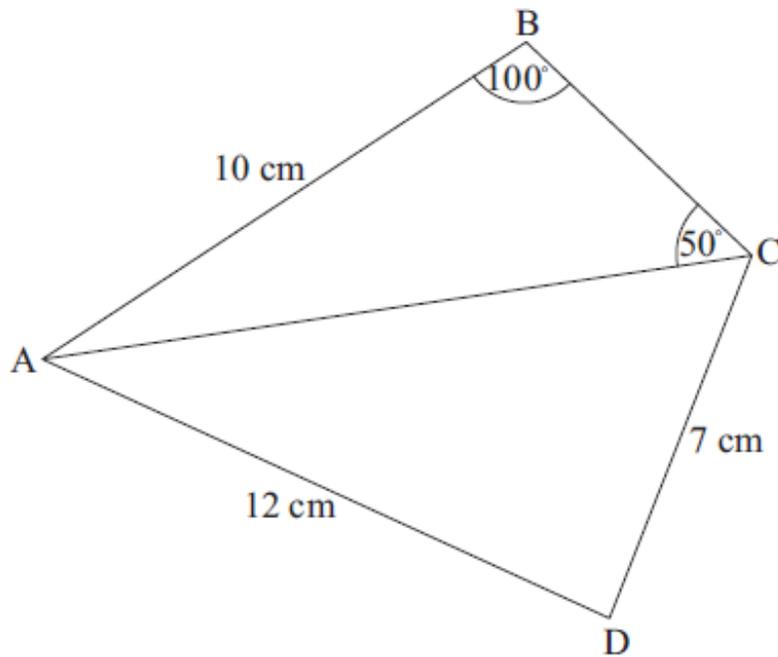
Write your answer to **part (b)(ii)** in the form $a \times 10^k$, where $1 \leq a < 10, k \in \mathbb{Z}$.

7a. [3 marks]

The quadrilateral ABCD has AB = 10 cm, AD = 12 cm and CD = 7 cm.

The size of angle ABC is 100° and the size of angle ACB is 50° .

diagram not to scale



Find the length of AC in centimetres.

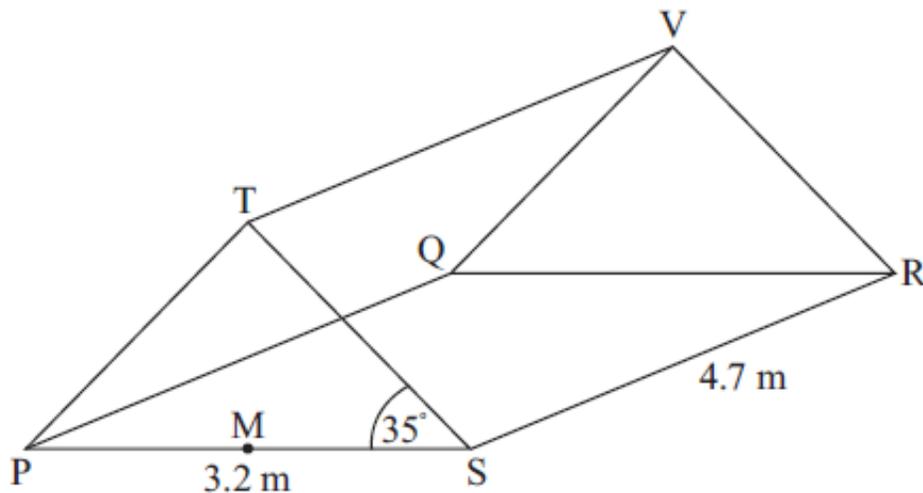
7b. [3 marks]

Find the size of angle ADC.

8a. [3 marks]

A tent is in the shape of a triangular right prism as shown in the diagram below.

diagram not to scale



The tent has a rectangular base PQRS .

PTS and QVR are isosceles triangles such that $PT = TS$ and $QV = VR$.

PS is 3.2 m, SR is 4.7 m and the angle TSP is 35° .

Show that the length of side ST is 1.95 m, correct to 3 significant figures.

8b. [3 marks]

Calculate the area of the triangle PTS.

8c. [1 mark]

Write down the area of the rectangle STVR.

8d. [3 marks]

Calculate the **total** surface area of the tent, including the base.

8e. [2 marks]

Calculate the volume of the tent.

8f. [4 marks]

A pole is placed from V to M, the midpoint of PS.

Find in metres,

(i) the height of the tent, TM;

(ii) the length of the pole, VM.

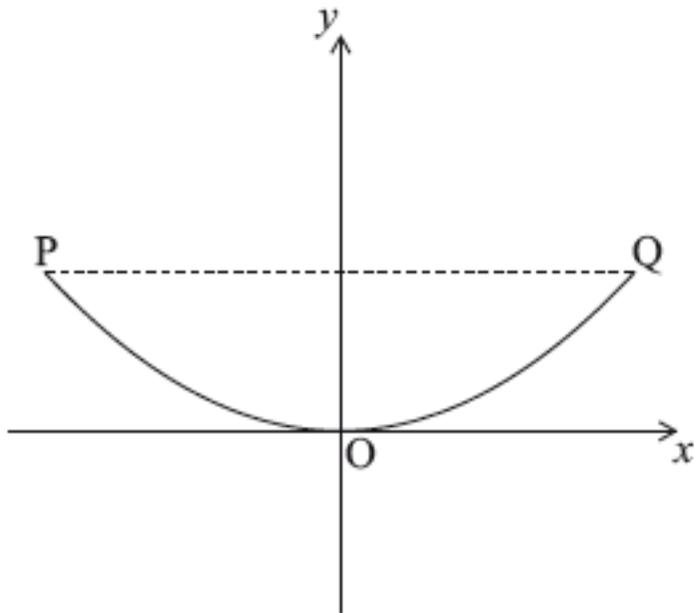
8g. [2 marks]

Calculate the angle between VM and the base of the tent.

9a. [1 mark]

The front view of the edge of a water tank is drawn on a set of axes shown below.

The edge is modelled by $y = ax^2 + c$.



Point **P** has coordinates $(-3, 1.8)$, point **O** has coordinates $(0, 0)$ and point **Q** has coordinates $(3, 1.8)$.

Write down the value of c .

9b. [2 marks]

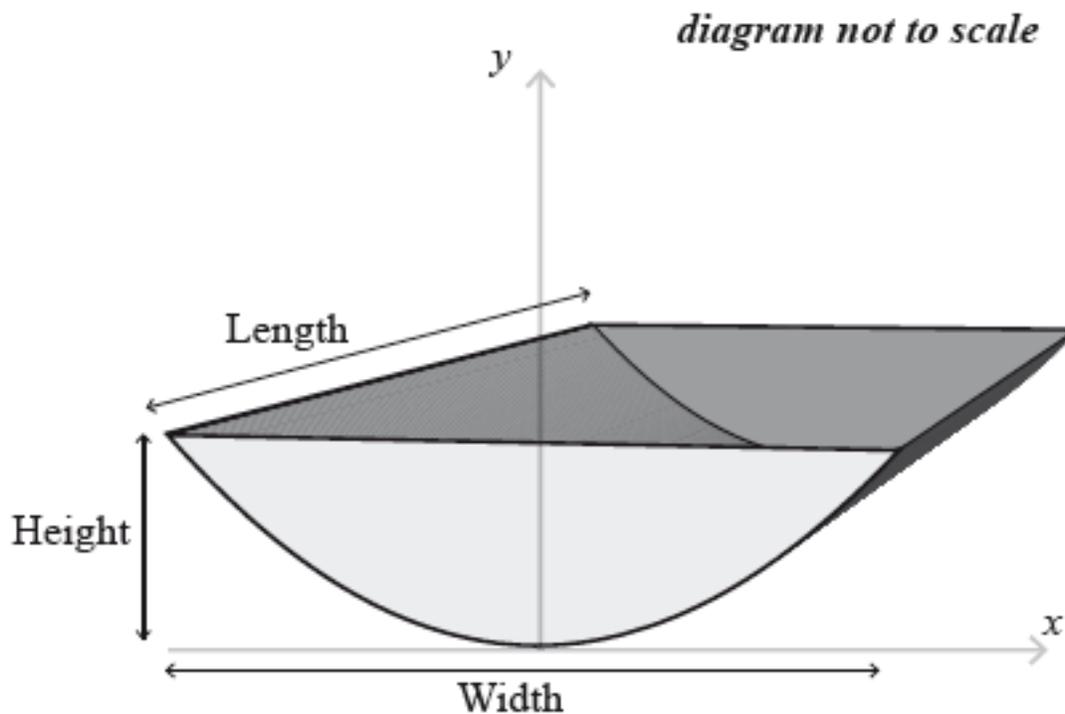
Find the value of a .

9c. [1 mark]

Hence write down the equation of the quadratic function which models the edge of the water tank.

9d. [2 marks]

The water tank is shown below. It is partially filled with water.



Calculate the value of y when $x = 2.4\text{m}$.

9e. [2 marks]

The water tank is shown below. It is partially filled with water.

State what the value of x and the value of y represent for this water tank.

9f. [2 marks]

The water tank is shown below. It is partially filled with water.

Find the value of x when the height of water in the tank is 0.9 m .

9g. [2 marks]

The water tank is shown below. It is partially filled with water.

When the water tank is filled to a height of 0.9 m , the front cross-sectional area of the water is 2.55 m^2 .

(i) Calculate the volume of water in the tank.

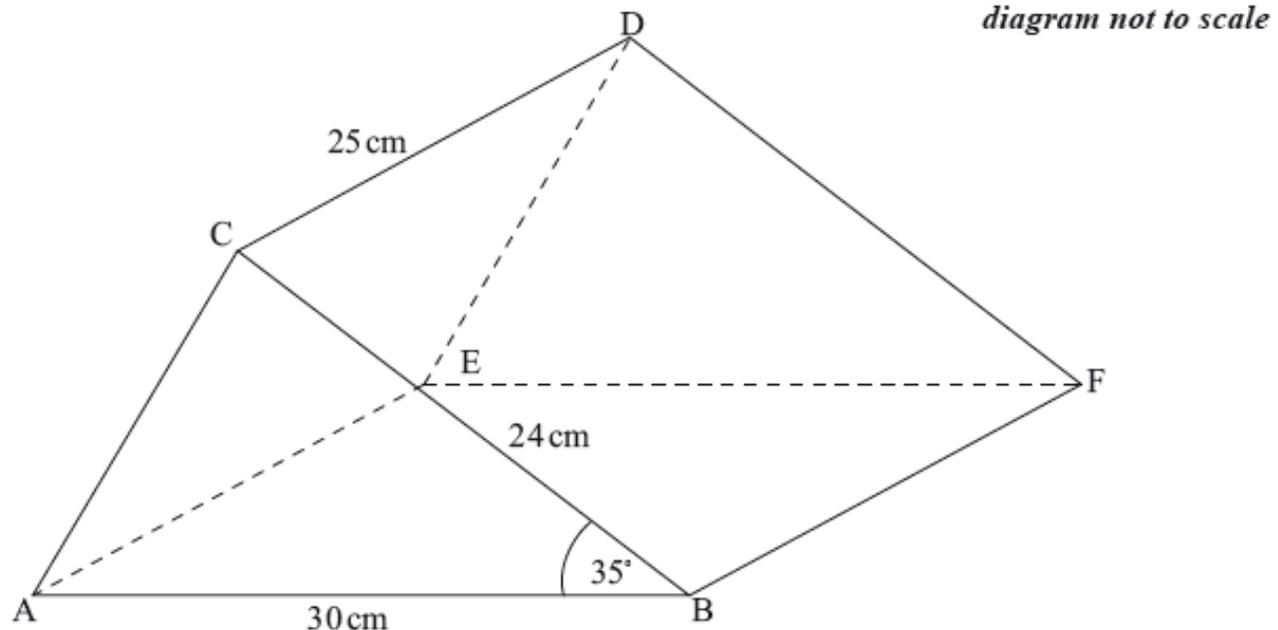
The total volume of the tank is 36 m^3 .

(ii) Calculate the percentage of water in the tank.

10a. [3 marks]

A manufacturer has a contract to make **2600** solid blocks of wood. Each block is in the shape of a right triangular prism, **ABCDEF**, as shown in the diagram.

AB = 30 cm, BC = 24 cm, CD = 25 cm and angle $\hat{A}BC = 35^\circ$.



Calculate the length of **AC**.

10b. [3 marks]

Calculate the area of triangle **ABC**.

10c. [2 marks]

Assuming that no wood is wasted, show that the volume of wood required to make all **2600** blocks is **$13\,400\,000 \text{ cm}^3$** , correct to three significant figures.

10d. [2 marks]

Write **$13\,400\,000$** in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

10e. [3 marks]

Show that the total surface area of one block is 2190 cm^2 , correct to three significant figures.

10f. [3 marks]

The blocks are to be painted. One litre of paint will cover 22 m^2 .

Calculate the number of litres required to paint all **2600** blocks.

11a. [3 marks]

ABC is a triangular field on horizontal ground. The lengths of AB and AC are 70 m and 50 m respectively. The size of angle BCA is 78° .

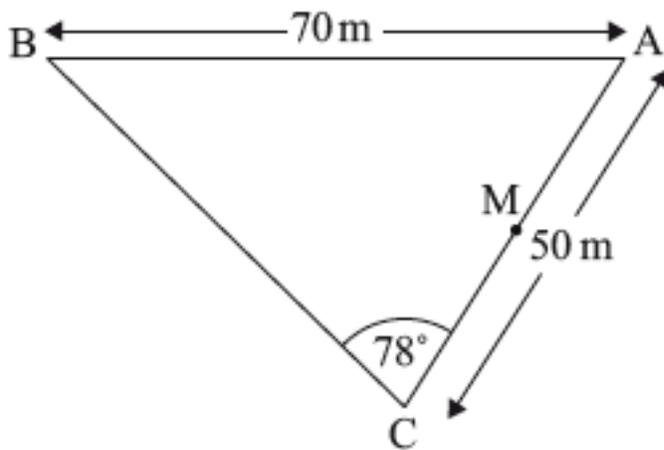


diagram not to scale

Find the size of angle ABC .

11b. [4 marks]

Find the area of the triangular field.

11c. [3 marks]

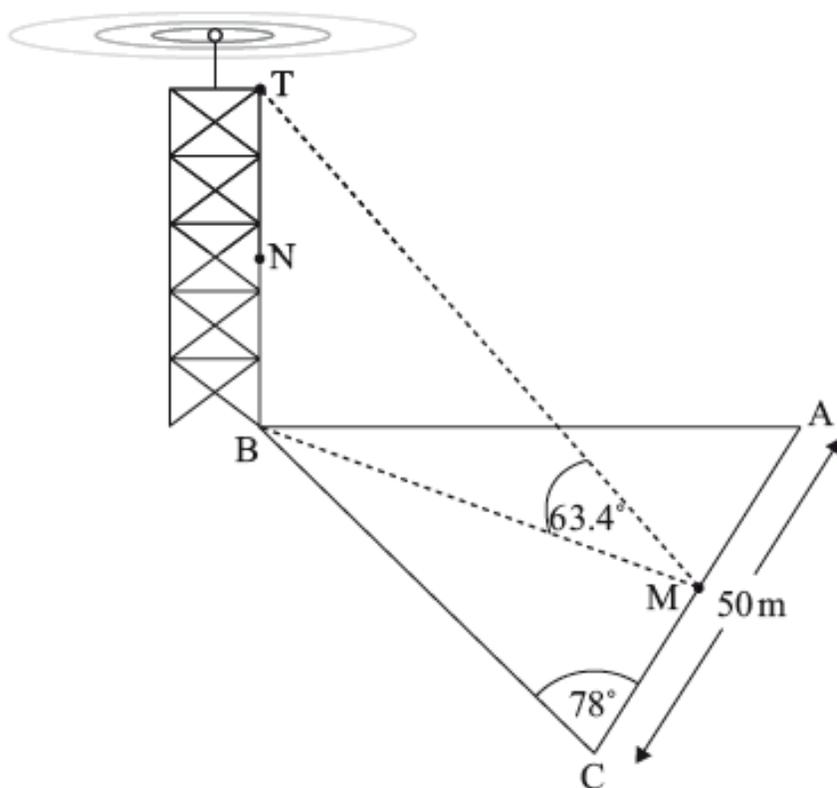
M is the midpoint of **AC**.

Find the length of **BM**.

11d. [5 marks]

A vertical mobile phone mast, **TB**, is built next to the field with its base at **B**. The angle of elevation of **T** from **M** is 63.4° . **N** is the midpoint of the mast.

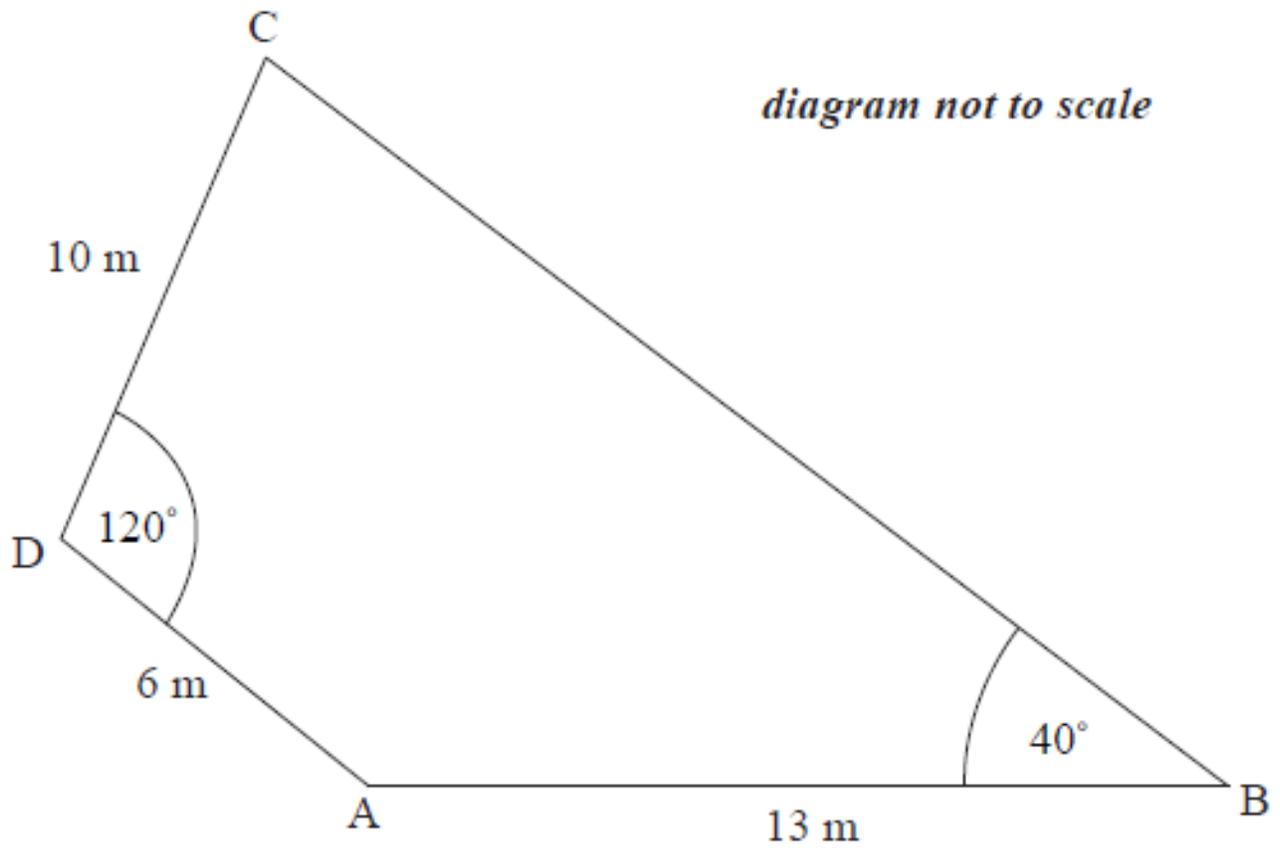
diagram not to scale



Calculate the angle of elevation of **N** from **M**.

12a. [3 marks]

The diagram shows quadrilateral ABCD in which $AB = 13\text{ m}$, $AD = 6\text{ m}$ and $DC = 10\text{ m}$. Angle $ADC = 120^\circ$ and angle $ABC = 40^\circ$.



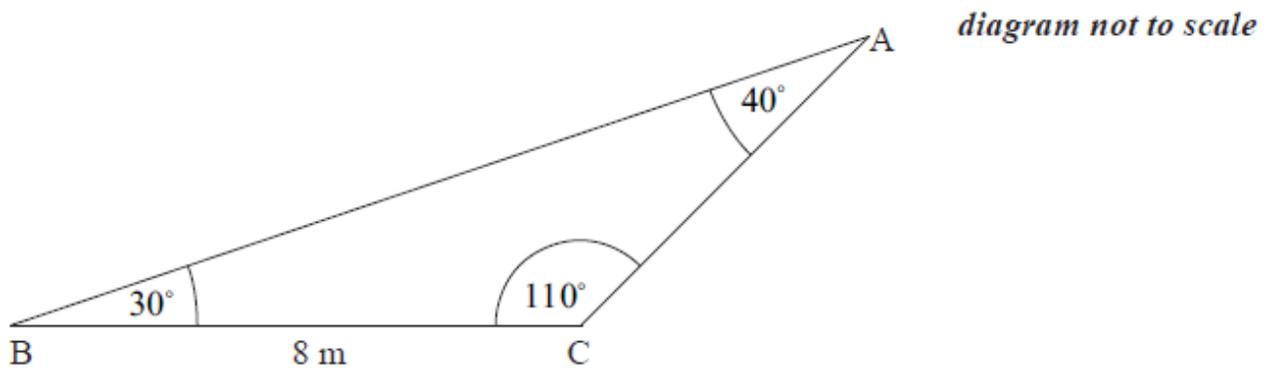
Calculate the length of AC.

12b. [3 marks]

Calculate the size of angle ACB.

13a. [3 marks]

In triangle ABC, BC = 8 m, angle ACB = 110° , angle CAB = 40° , and angle ABC = 30° .



Find the length of AC.

13b. [3 marks]

Find the area of triangle ABC.

14a. [1 mark]

In the diagram, triangle ABC is isosceles. $AB = AC$ and angle ACB is 32° . The length of side AC is x cm.

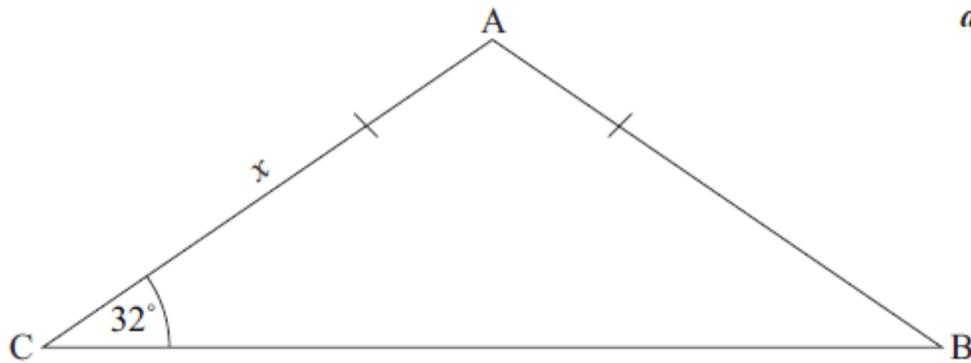


diagram not to scale

Write down the size of angle CBA.

14b. [1 mark]

Write down the size of angle CAB.

14c. [4 marks]

The area of triangle ABC is 360 cm. Calculate the length of side AC. Express your answer in **millimetres**.

15a. [3 marks]

The diagram shows triangle ABC in which $AB = 28$ cm, $BC = 13$ cm, $BD = 12$ cm and $AD = 20$ cm.

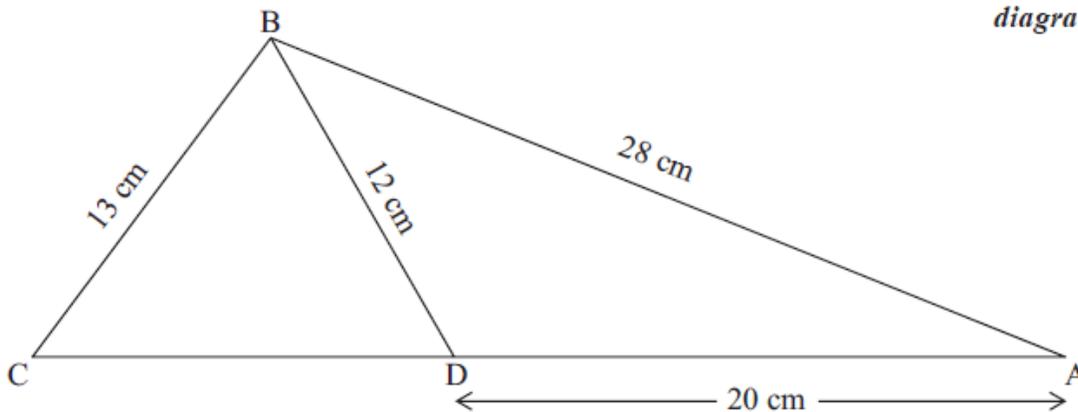


diagram not to scale

Calculate the size of angle ADB.

15b. [3 marks]

Find the area of triangle ADB.

15c. [4 marks]

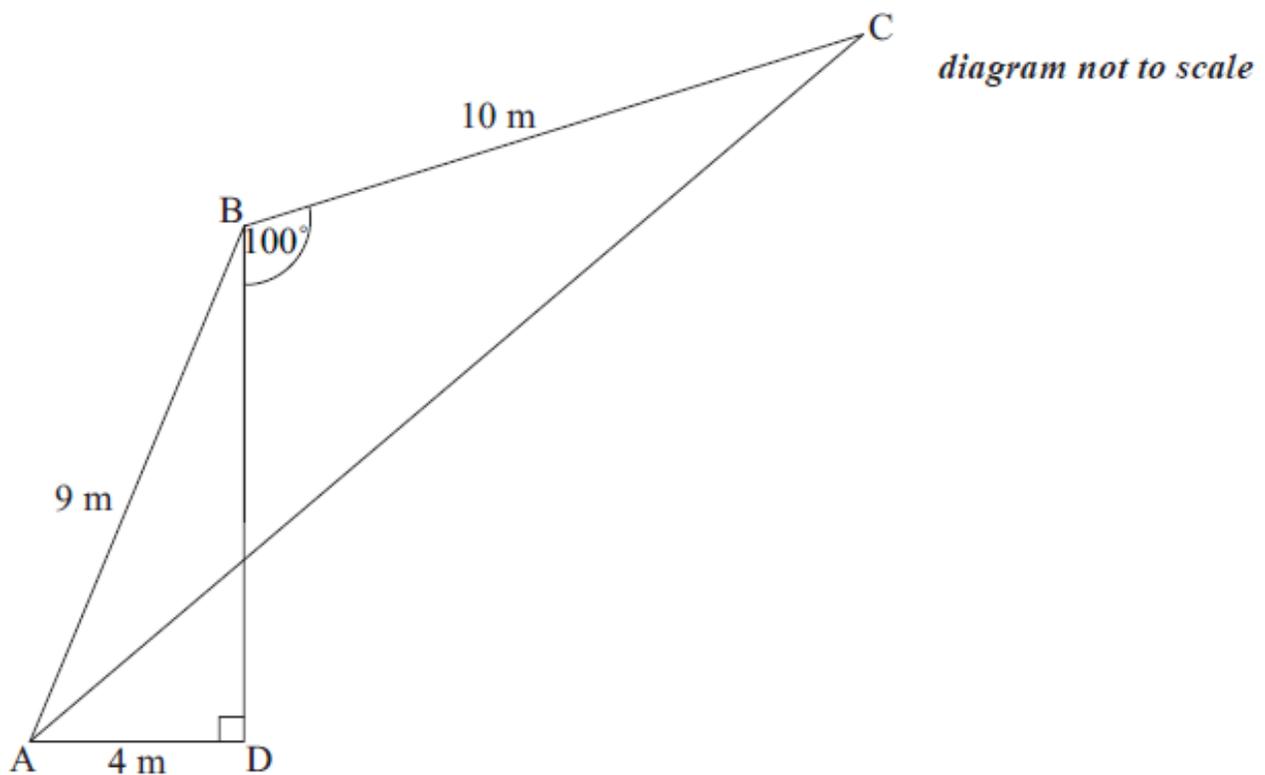
Calculate the size of angle BCD.

15d. [4 marks]

Show that the triangle ABC is not right angled.

16a. [3 marks]

In the diagram, $AD = 4\text{m}$, $AB = 9\text{m}$, $BC = 10\text{m}$, $\hat{BDA} = 90^\circ$ and $\hat{DBC} = 100^\circ$.



Calculate the size of \hat{ABC} .

16b. [3 marks]

Calculate the length of AC.

17a. [1 mark]

The diagram shows a triangle ABC in which $AC = 17\text{ cm}$. M is the midpoint of AC. Triangle ABM is equilateral.

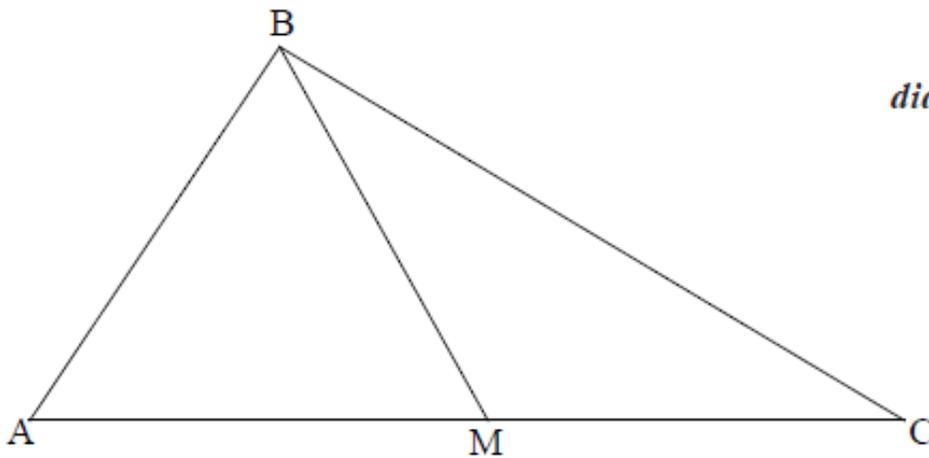


diagram not to scale

Write down the size of angle MCB.

17b. [1 mark]

Write down the length of BM in cm.

17c. [1 mark]

Write down the size of angle BMC.

17d. [3 marks]

Calculate the length of BC in cm.

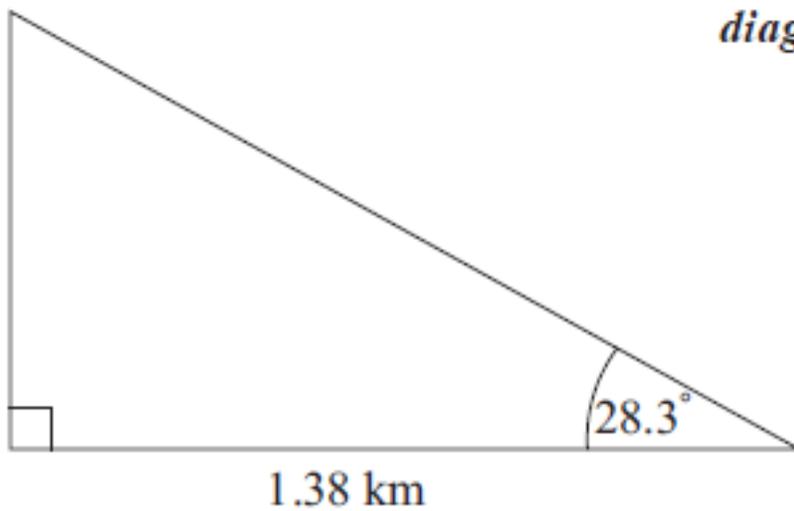
18a. [1 mark]

José stands 1.38 kilometres from a vertical cliff.

Express this distance in metres.

18b. [3 marks]

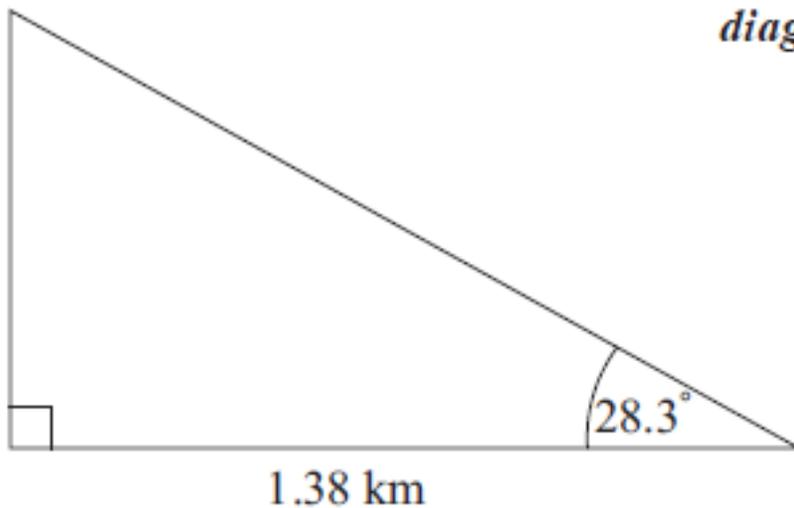
José estimates the angle between the horizontal and the top of the cliff as 28.3° and uses it to find the height of the cliff.



Find the height of the cliff according to José's calculation. **Express your answer in metres, to the nearest whole metre.**

18c. [2 marks]

José estimates the angle between the horizontal and the top of the cliff as 28.3° and uses it to find the height of the cliff.

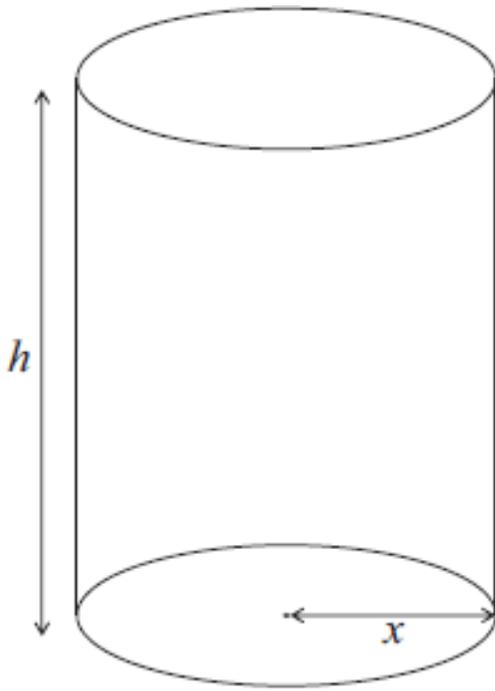


The actual height of the cliff is 718 metres. Calculate the percentage error made by José when calculating the height of the cliff.

19a. [2 marks]

A dog food manufacturer has to cut production costs. She wishes to use as little aluminium as possible in the construction of cylindrical cans. In the following diagram, h represents the height of the can in cm and x , the radius of the base of the can in cm.

diagram not to scale



The volume of the dog food cans is 600 cm.

Show that $h = \frac{600}{\pi x^2}$.

19b. [2 marks]

Find an expression for the curved surface area of the can, in terms of x . Simplify your answer.

19c. [2 marks]

Hence write down an expression for A , the total surface area of the can, in terms of x .

19d. [3 marks]

Differentiate A in terms of x .

19e. [3 marks]

Find the value of x that makes A a minimum.

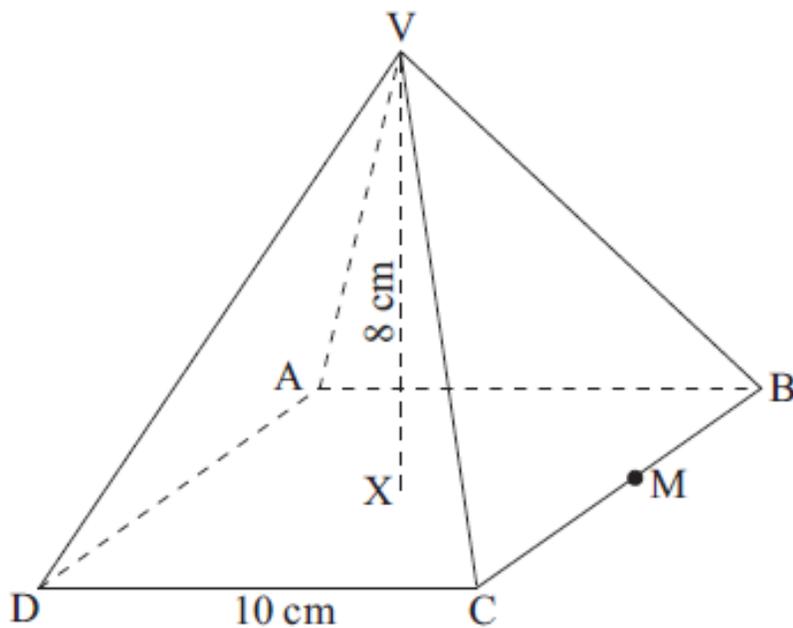
19f. [2 marks]

Calculate the minimum total surface area of the dog food can.

20a. [1 mark]

The diagram below shows a square based right pyramid. ABCD is a square of side 10 cm. VX is the perpendicular height of 8 cm. M is the midpoint of BC.

diagram not to scale



Write down the length of XM.

20b. [1 mark]

In a mountain region there appears to be a relationship between the number of trees growing in the region and the depth of snow in winter. A set of 10 areas was chosen, and in each area the number of trees was counted and the depth of snow measured. The results are given in the table below.

Number of trees (x)	Depth of snow in cm (y)
45	30
75	50
66	40
27	25
44	30
28	5
60	35
35	20
73	45
47	25

Use your graphic display calculator to find the standard deviation of the number of trees.

20c. [2 marks]

Calculate the length of VM.

20d. [2 marks]

Calculate the angle between VM and ABCD.

20e. [4 marks]

A path goes around a forest so that it forms the three sides of a triangle. The lengths of two sides are 550 m and 290 m. These two sides meet at an angle of 115° . A diagram is shown below.

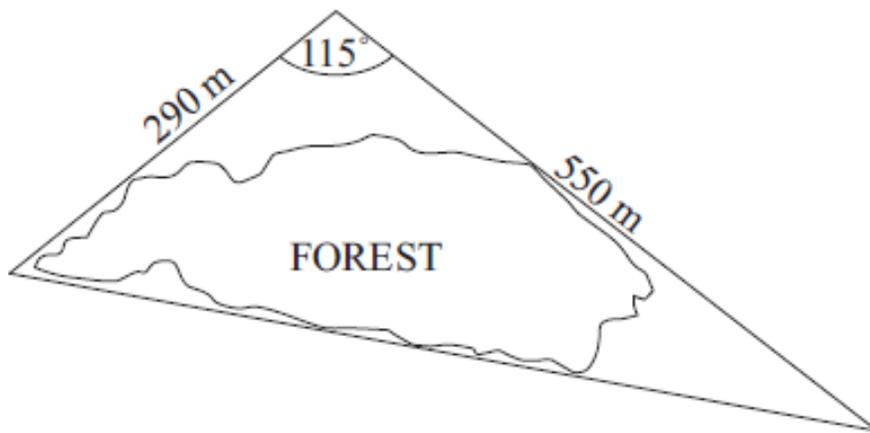


diagram not to scale

Calculate the length of the third side of the triangle. Give your answer correct to the nearest 10 m.

20f. [3 marks]

Calculate the area enclosed by the path that goes around the forest.

20g. [4 marks]

Inside the forest a second path forms the three sides of another triangle named ABC. Angle BAC is 53° , AC is 180 m and BC is 230 m.

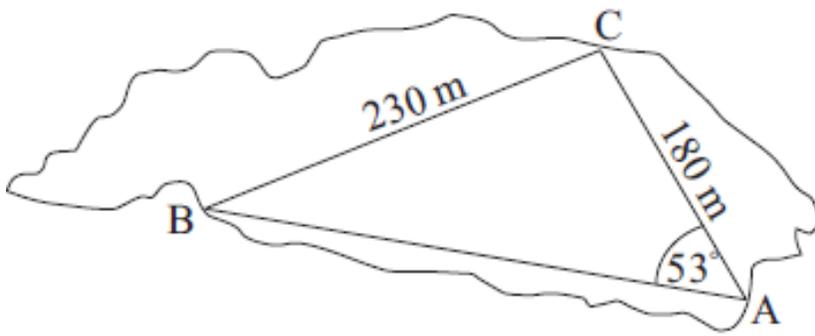


diagram not to scale

Calculate the size of angle ACB.

21a. [3 marks]

Pauline owns a piece of land ABCD in the shape of a quadrilateral. The length of BC is **190m** , the length of CD is **120m** , the length of AD is **70m** , the size of angle BCD is **75°** and the size of angle BAD is **115°**

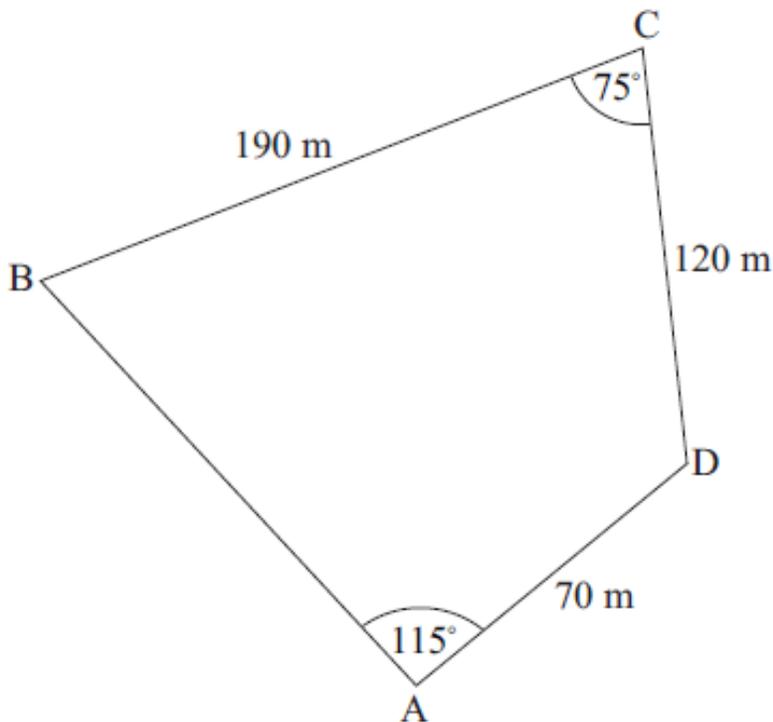


diagram not to scale

Pauline decides to sell the triangular portion of land ABD . She first builds a straight fence from B to D .

Calculate the length of the fence.

21b. [2 marks]

The fence costs **17** USD per metre to build.

Calculate the cost of building the fence. Give your answer correct to the nearest USD.

21c. [3 marks]

Show that the size of angle ABD is 18.8° , correct to three significant figures.

21d. [4 marks]

Calculate the area of triangle ABD.

21e. [2 marks]

She sells the land for **120** USD per square metre.

Calculate the value of the land that Pauline sells. Give your answer correct to the nearest USD.

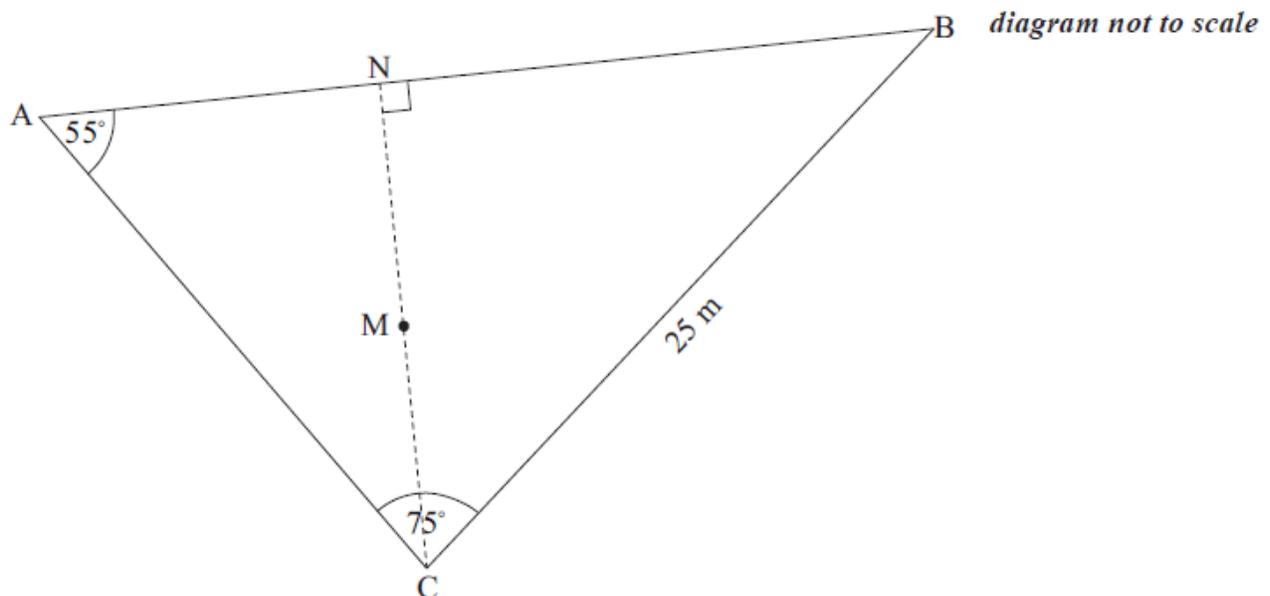
21f. [4 marks]

Pauline invests **300000** USD from the sale of the land in a bank that pays compound interest compounded annually.

Find the interest rate that the bank pays so that the investment will double in value in 15 years.

22a. [1 mark]

The diagram represents a small, triangular field, ABC, with **BC = 25m**, **angle BAC = 55°** and **angle ACB = 75°** .



Write down the size of angle ABC.

22b. [3 marks]

Calculate the length of AC.

22c. [3 marks]

Calculate the area of the field ABC.

22d. [3 marks]

N is the point on AB such that CN is perpendicular to AB. M is the midpoint of CN.

Calculate the length of NM.

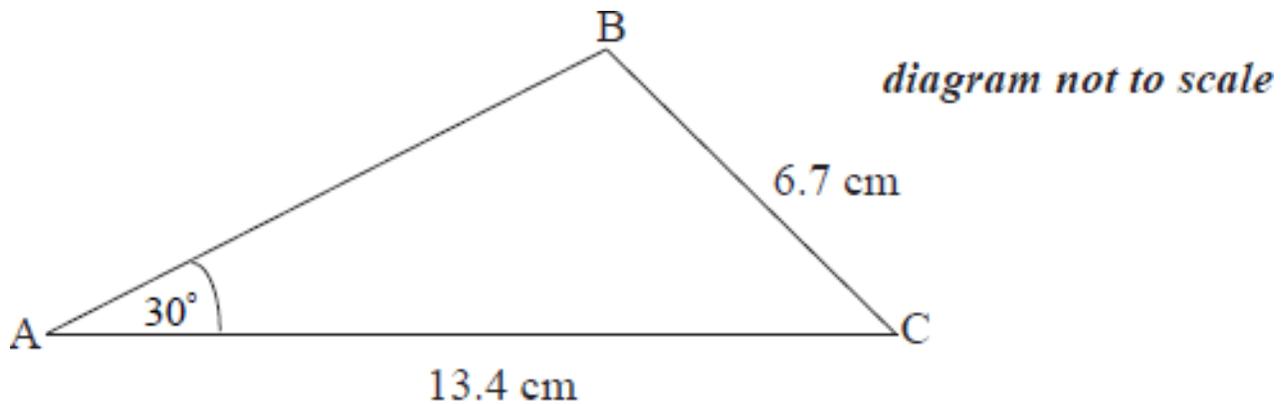
22e. [5 marks]

A goat is attached to one end of a rope of length 7 m. The other end of the rope is attached to the point M.

Decide whether the goat can reach point P, the midpoint of CB. Justify your answer.

23a. [4 marks]

The diagram shows triangle ABC in which angle BAC = 30° , BC = 6.7 cm and AC = 13.4 cm.



Calculate the size of angle ACB.

23b. [2 marks]

Nadia makes an accurate drawing of triangle ABC. She measures angle BAC and finds it to be 29° .

Calculate the percentage error in Nadia's measurement of angle BAC.

24a. [3 marks]

The quadrilateral ABCD shown below represents a sandbox. AB and BC have the same length. AD is **9m** long and CD is **4.2m** long. Angles ADC and ABC are **95°** and **130°** respectively.

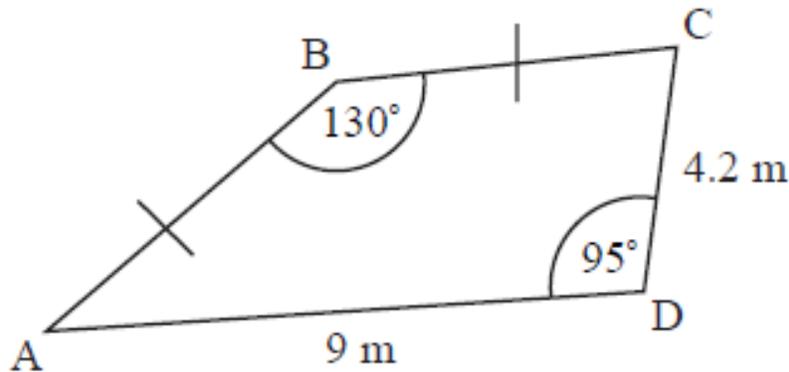


diagram not to scale

Find the length of AC.

24b. [4 marks]

- (i) Write down the size of angle BCA.
- (ii) Calculate the length of AB.

24c. [4 marks]

Show that the area of the sandbox is **31.1 m²** correct to 3 s.f.

24d. [3 marks]

The sandbox is a prism. Its edges are **40 cm** high. The sand occupies one third of the volume of the sandbox. Calculate the volume of sand in the sandbox.

25a. [1 mark]

Triangle **ABC** is such that **AC** is **7 cm**, angle **ABC** is **65°** and angle **ACB** is **30°**.

Sketch the triangle writing in the side length and angles.

25b. [2 marks]

Calculate the length of **AB**.

25c. [3 marks]

Find the area of triangle **ABC**.

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