Geometry 1

1a) $150 \times 92 = 13800\ cm^2$

b) $75\ cm$

c) $13800 \div 3 = 4600\ cm^2$

d) $A_A = \frac{75(x+92)}{2} = 4600$

$A_A = 75(x+92) = 9200$

$A_A = x + 92 = 122.66$

$x = 30.7\ cm$

2a) $A = 2x(x-4)$

$A = 2x^2 - 8x$

b) $10 = 2x^2 - 8x$

$2x^2 - 8x - 10 = 0$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

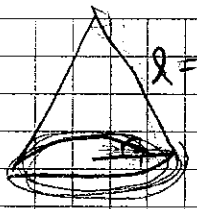
$x_1 = 5$ ⇐

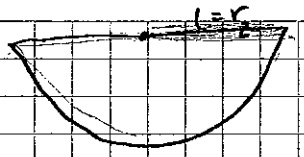
$(x_2 = -1)$

c) Length $2 \times 5 = 10\ m$

Width $5 - 4 = 1\ m$

3a)





$A_{circle} = \pi r^2$

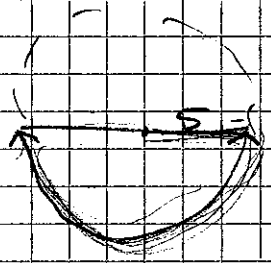
$A_{semicircle} = \frac{\pi r^2}{2}$

$39.27 = \frac{\pi r^2}{2}$

$78.54 = \pi r^2$

$25.00 = r^2$

$r_2 = 5$



3b) i) $C = 2\pi r$

$C_{semicircle} = \frac{2\pi 5}{2}$

$C = 15.7 \text{ m}$

ii) $C = 2\pi r$

$2\pi r = 15.70796...$

$r = 2.5 \text{ m}$

iii) $h^2 + 2.5^2 = 5^2$

$h^2 = 5^2 - 2.5^2$

$h = 4.33 \text{ m}$

9.33 − 2r

3c $2r + h = 9.33$

$h = 9.33 - 2r$

3d $V_{cone} = \frac{\pi r^2 h}{3}$ $\quad h = 9.33 - 2r$

$\Rightarrow \quad \frac{\pi r^2}{3} \times (9.33 - 2r)$

$\frac{9.33\pi r^2 - 2\pi r^3}{3} = 3.11\pi r^2 - \frac{2}{3}\pi r^3$

3e) $V' = 6.22\pi r - 6\frac{\pi r^2}{3}$

$V' = 6.22\pi r - 2\pi r^2$

f) i) $6.22\pi r - 2\pi r^2 = 0$

$2\pi r(3.11 + r) = 0$

$r_1 = 3.11$ ←

$(r_2 = 0)$

ii) $r = 3.11 \Rightarrow V = 3.11\pi \times 3.11^2 - \frac{2}{3}\pi \times 3.11^3$

$V = 31.5 \ m^3$

4a) $(0, 4)$

b) i) $(a, 4)$

ii) $m = \frac{4-0}{a-0} = \frac{4}{a}$

c) i) $-\frac{a}{4}$

ii)

4 c) ii) $m = -\frac{a}{4} \Rightarrow y = -\frac{a}{4}x + c$

Use p.t $(a, 4)$

$$4 = -\frac{a}{4} \times a + c$$

$$4 = -\frac{a^2}{4} + c$$

$$4 + \frac{a^2}{4} = c$$

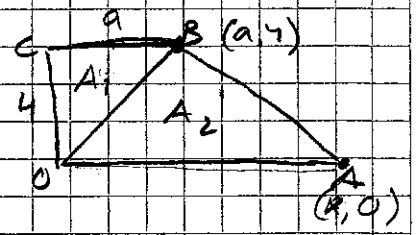
in $y = mx + c$

$$\Rightarrow y = -\frac{a}{4}x + 4 + \frac{a^2}{4}$$

$$y = -\frac{a}{4}x + \frac{1}{4}a^2 + 4 \quad (\times 4) \Rightarrow 4y = -ax + a^2 + 16$$

$$4y + ax - a^2 - 16 = 0$$

4 d) i) $A_1 = \frac{4a}{2} = 2a$



ii) $3 \times 2a = \frac{4x}{2}$

$$6a = \frac{4x}{2}$$

$$12a = 4x$$

$$x = 3a$$

e) Where $y = 0$ on the line $4y + ax - a^2 - 16 = 0$ $\quad x = 3a \Rightarrow$

$$4(0) + a \times 3a - a^2 - 16 = 0$$

$$3a^2 - a^2 - 16 = 0$$

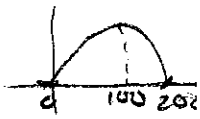
$$2a^2 - 16 = 0$$

$$2a^2 = 16$$

$$a^2 = 8$$

$$a = 2.83$$

6c) $A = x(200 - x)$ $(A = 200x - x^2)$ $\Rightarrow$



i) $A = 0 \Rightarrow x(200 - x) = 0$

$x_1 = 0$

$x_2 = 200$

Equation of axis of symmetry

$x = 100$

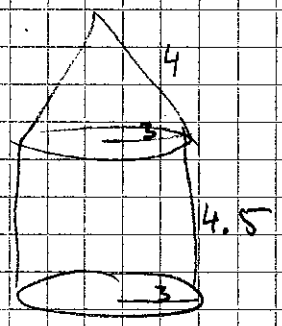
ii) When $x = 100 \Rightarrow$

$A = 100(200 - 100)$

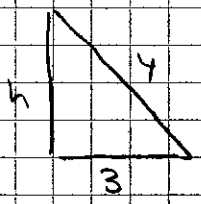
$A = 100 \times 100 = 10000 \ m^2$

6d) $3600 \leq A(x) \leq 10000$ Be aware of the domain!

7a)



a) i)



$3^2 + h^2 = 4^2$

$h^2 = 16 - 9$

$h^2 = 7$

$h = \sqrt{7}$

$h \approx 2.65$

5a) $AC^2 = 8^2 + 6^2$

$64 + 36 = AC^2$

$100 = AC^2$

$AC = 10 \Rightarrow AH = 5$

$5^2 + VH^2 = 13^2$

$25 + VH^2 = 169$

$VH^2 = 169 - 25$

$VH^2 = 144$

$VH = 12$ cm

5b $V_p = \frac{Ah}{3}$

$V_p = \frac{8 \times 6 \times 12}{3} = 192 \text{ cm}^3$

6a) $x = 20 \Rightarrow A = 20(200 - 20)$

$A = 20 \times 180$

$A = 3600 \text{ m}^2$

b) $x(200 - x) = 3600$

$200x - x^2 = 3600$

$x^2 - 200x + 3600 = 0$

$x_1 = 180$

$x_2 = 20$

7a) ii) $V_{cyl} = \pi r^2 h$

$V_{cone} = \frac{\pi r^2 h}{3}$

$(\pi \times 3^2 \times 4.5) + \frac{(\pi \times 3^2 \times V7)}{3} = 152\ cm^3$

7b) $\pi \times 3^2 \times h = 125$

$h = \frac{125}{9\pi}$

$h = 4.42$

7c) Surface area cone: $\pi r l$

Surface are cylinder $2\pi rh + \pi r^2$

curved, bottom

$(\pi \times 3 \times 4) + (2\pi \times 3 \times 4.5) + (\pi \times 3^2)$

cone, curved cyl, bottom cyl

$= 151\ cm^2$

7d) $\frac{150.796...}{7} = 21.54...$ millilitre

$21.542... \times 3 = 64.63$ ZAR

7e) 1 EUR = 13.03 ZAR

$\frac{1}{13.03}$ EUR = 1 ZAR

$\frac{325}{13.03}$ EUR = 325 ZAR

24.94 EUR = 325 ZAR

8 a) $A = (x+1)^2 - 1 + 1$ $\quad (A = (x+1)^2 - 1)$

$A = x^2 + 2x + 1 - 1$

$A = x^2 + 2x$ (wrong in markscheme)

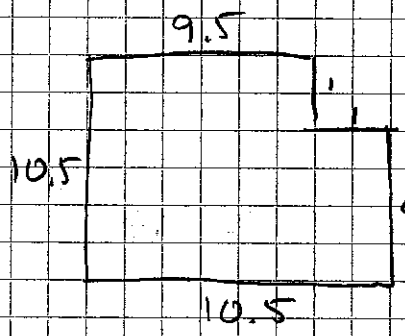
b) $x^2 + 2x = 109.25$

$x^2 + 2x - 109.25 = 0$

$(x_1 = +11.5)$

$x_2 = 9.5$

c)



42 m

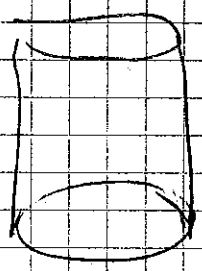
9 a) $\pi r^2 = 8$

$r^2 = \frac{8}{\pi}$

$r = \sqrt{\frac{8}{\pi}}$

$r = 1.60$ cm

b)



$V = \pi \sqrt{\frac{8}{\pi}}^2 \times 25$

$V = 200 \text{ cm}^3$

Note; Wrong in markscheme

9 c) surface area

$2 \times 8 + 2 \times 11 \times \sqrt{\frac{8}{11}} \times 25$

$= 267 \text{ cm}^2$ Note the unit!

10 a) $AB^2 + 8^2 = 9.5^2$

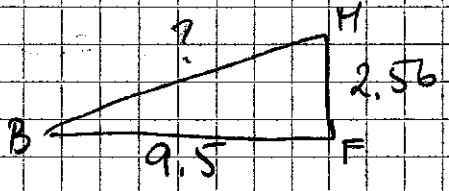
$AB^2 = 90.25 - 64$

$AB^2 = 26.25$

$AB = 5.12 \text{ cm}$

b) $AB = AD = EF$ $EF = 5.12$

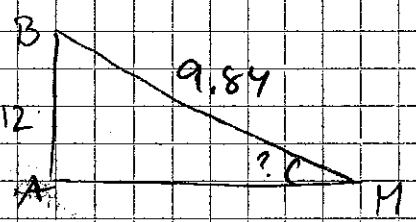
$\Rightarrow FM = \frac{5.12}{2} = 2.56$



$2.56^2 + 9.5^2 = BM^2$

$96.8125 = BM^2$

$BM = 9.84$

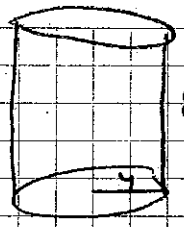


c) $\sin\theta = \frac{opp}{hyp}$

$\sin\theta = \frac{5.12}{9.84}$

$\theta = 31.4°$

11 a)



a) πr^2 Note: The unit

$\Rightarrow \pi \times 4^2 = 50.3\ cm^2$

b) Base area × height $\Rightarrow$ $50.265... \times 8 =$

$= 402.12...\ cm^3$

$\approx 402\ cm^3$

c) $\pi r^2 + \pi r^2 + 2\pi rh$

$\Rightarrow \underbrace{50.265... + 50.265...}_{(50.265 \times 2)} + 2\pi \times 4 \times 8$

$= 301.59... \approx 302\ cm^2$ Note the unit!

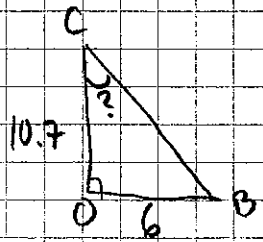
d) $\text{Volume}_{cone} = \frac{\pi r^2 h}{3} \Rightarrow \frac{\pi \times 6^2 \times h}{3} = 402.12...$

$\pi \times 36 \times h = 1206.37...$

$h = \frac{1206.37...}{(36\pi)}$

$h = 10.7\ cm$

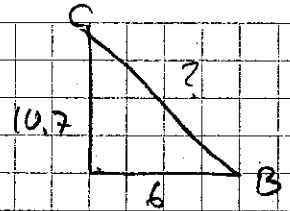
e)



$\tan\theta = \frac{6}{10.7}$

$\theta = 29.4°$

11f)



$6^2 + 10.86^2 = CB^2$

$CB = 12.2$ cm

g) $\pi rl + \pi r^2 \Rightarrow \pi \times 6 \times 12.2 + \pi \times 6^2$

$= 344 \text{ cm}^2$ UNIT!

12a) $4x + 4y + 4 \times 2x = 48$

$4x + 4y + 8x = 48$

$4y + 12x = 48$

$4y = 48 - 12x$

$y = 12 - 3x$

b) Base area × height

$\Rightarrow V = 2x \times x \times (12 - 3x)$

$V = 2x^2(12 - 3x)$

$V = 24x^2 - 6x^3$ Note: exponents,

c) $V' = 48x - 18x^2$

d) Max volume when $V' = 0$

$48x - 18x^2 = 0$

$18x^2 - 48x = 0$

$6x(3x - 8) = 0$

$(x_1 = 0)$

$x_2 = 2.67$ m ←

$3x - 8 = 0$

$3x = 8$

$x = \frac{8}{3}$

$x = 2.67$

12 e) $V = 24x^2 + 6x^3 \Rightarrow$

$V = 24 \times 2.67^2 - 6 \times 2.67^3$

$V = 56.9\ m^3$

f) $L = 2x \Rightarrow 5.33\ m$

$h = 12 - 3x \Rightarrow 12 - 3 \times 2.67 = 4\ m$

g) Surface area

$2 \times 2.66 \times 4 + 2 \times 5.33 \times 4 + 2 \times 5.33 \times 2.66$

$= 92.4\ m^2$ (for 2.66 you can use $(\frac{8}{3})$ in the calculator)

13a) Radius = 150 000 000

Distance each year:

$C = 2\pi r \Rightarrow 2\pi \times 150\,000\,000$

$= 942477796.1$

Per day:

$942477796.1 / 365 = 2582137$

$\Rightarrow$ answer 25 800 00 km

b) 2.58×10^6

14 a) $x + (x+7)^2 = (x+8)^2$

$x^2 + x^2 + 14x + 49 = x^2 + 16x + 64$

$2x^2 + 14x + 49 = x^2 + 16x + 64$

$x^2 - 2x - 15 = 0$

b) $x^2 - 2x + 15 = 0$

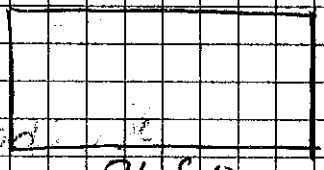
$(x-5)(x+3) = 0$

$x_1 = 5$

$(x_2 = -3)$

c) $5 + 12 + 13 = 30$ cm

15 a)



9300 cm

9.3×10^3

b) $2680 \times 1970 = 5279600$ cm^2

$\Rightarrow 5280000$

16 a)