Geometry 1, DP2

1a. *[1 mark]*

The diagram below represents a rectangular flag with dimensions 150 cm by 92 cm. The flag is divided into three regions A, B and C.

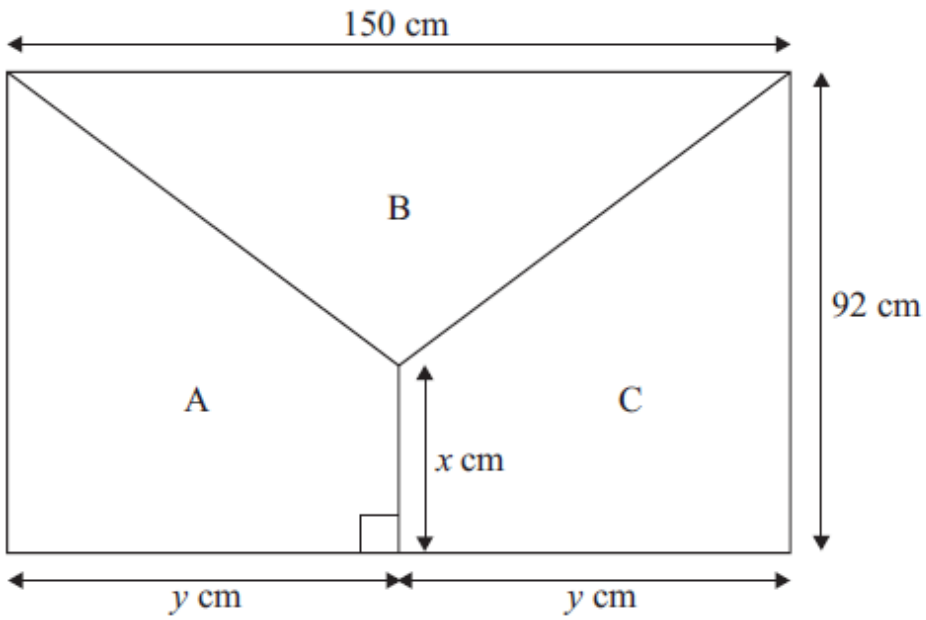


diagram not to scale

Write down the total area of the flag.

1b. *[1 mark]*

Write down the value of y.

1c. *[1 mark]*

The areas of regions A, B, and C are equal.

Write down the area of region A.

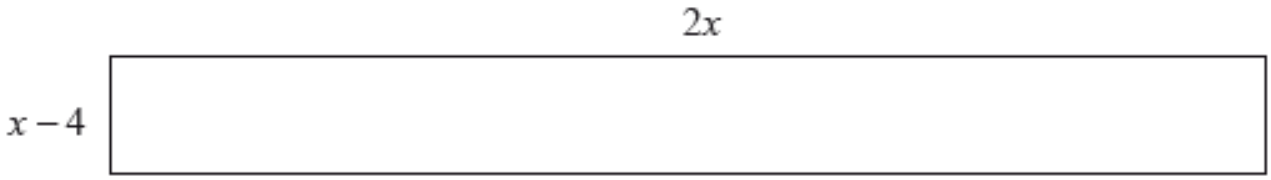
1d. *[3 marks]*

Using your answers to **parts (b) and (c)**, find the value of x.

2a. *[1 mark]*

The surface of a red carpet is shown below. The dimensions of the carpet are in metres.

diagram not to scale



Write down an expression for the area, A, in m^2, of the carpet.

2b. *[3 marks]*

The area of the carpet is $10\ \text{m}^2$.

Calculate the value of x.

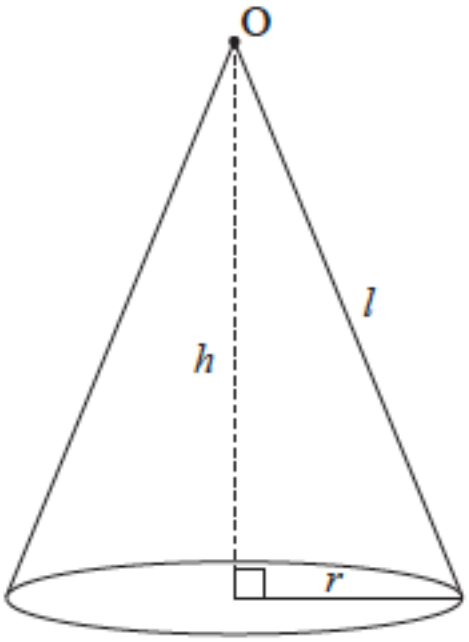
2c. *[2 marks]*

The area of the carpet is $10\ \text{m}^2$.

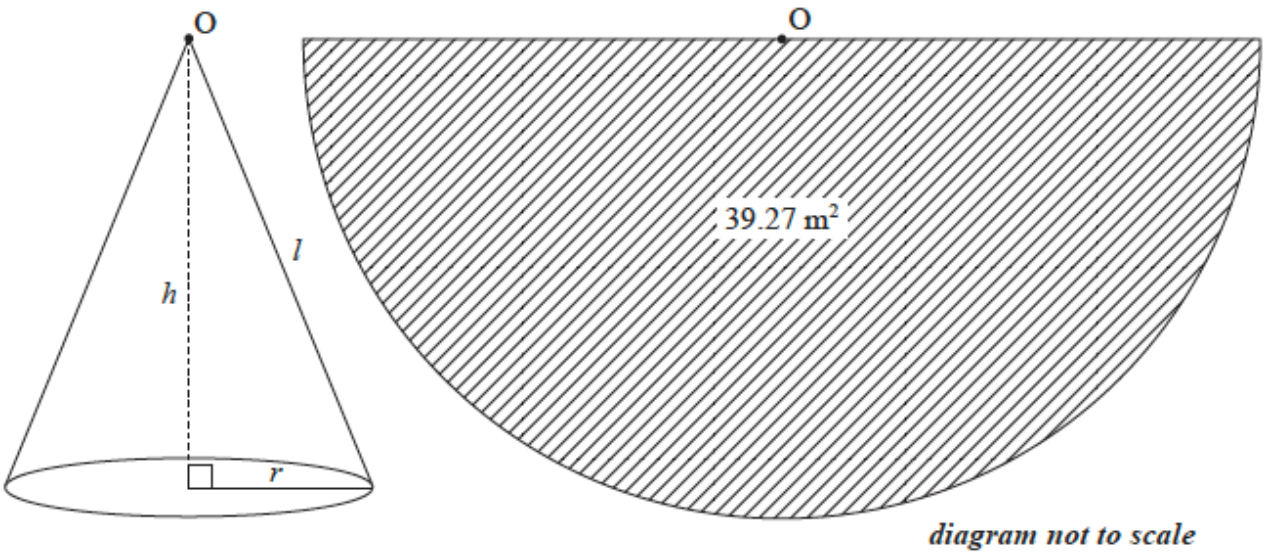
Hence, write down the value of the length and of the width of the carpet, in metres.

3a. *[2 marks]*

Tepees were traditionally used by nomadic tribes who lived on the Great Plains of North America. They are cone-shaped dwellings and can be modelled as a cone, with vertex O, shown below. The cone has radius, r, height, h, and slant height, l.



A model tepee is displayed at a Great Plains exhibition. The curved surface area of this tepee is covered by a piece of canvas that is $39.27\ \text{m}^2$, and has the shape of a semicircle, as shown in the following diagram.



Show that the slant height, l, is 5 m, correct to the nearest metre.

3b. *[6 marks]*

(i) Find the circumference of the base of the cone.

(ii) Find the radius, r, of the base.

(iii) Find the height, h.

3c. *[1 mark]*

A company designs cone-shaped tents to resemble the traditional tepees.

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Write down an expression for the height, h, in terms of the radius, r, of these cone-shaped tents.

3d. *[1 mark]*

A company designs cone-shaped tents to resemble the traditional tepees.

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Show that the volume of the tent, V, can be written as

$$V = 3.11\pi r^2 - \frac{2}{3}\pi r^3.$$

3e. *[2 marks]*

A company designs cone-shaped tents to resemble the traditional tepees.

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Find $\frac{dV}{dr}$.

3f. *[4 marks]*

A company designs cone-shaped tents to resemble the traditional tepees.

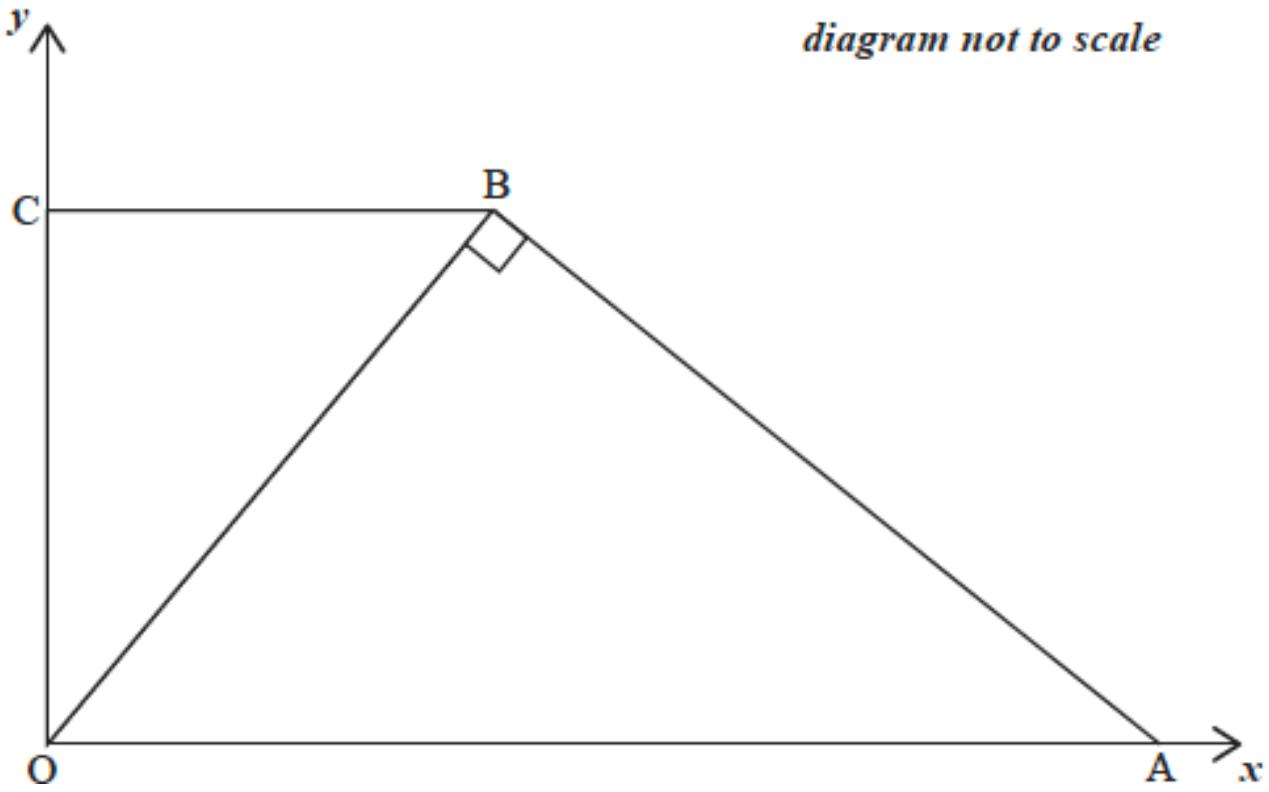
These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

(i) Determine the exact value of r for which the volume is a maximum.

(ii) Find the maximum volume.

4a. *[1 mark]*

The following diagram shows two triangles, OBC and OBA, on a set of axes. Point C lies on the y-axis, and O is the origin.



The equation of the line BC is $y = 4$.

Write down the coordinates of point C.

4b. *[2 marks]*

The x-coordinate of point B is a.

(i) Write down the coordinates of point B;

(ii) Write down the gradient of the line OB.

4c. *[4 marks]*

Point A lies on the x-axis and the line AB is perpendicular to line OB.

(i) Write down the gradient of line AB.

(ii) Show that the equation of the line AB is $4y + ax - a^2 - 16 = 0$.

4d. *[3 marks]*

The area of triangle AOB is **three times** the area of triangle OBC.

Find an expression, **in terms of** a, for

(i) the area of triangle OBC;

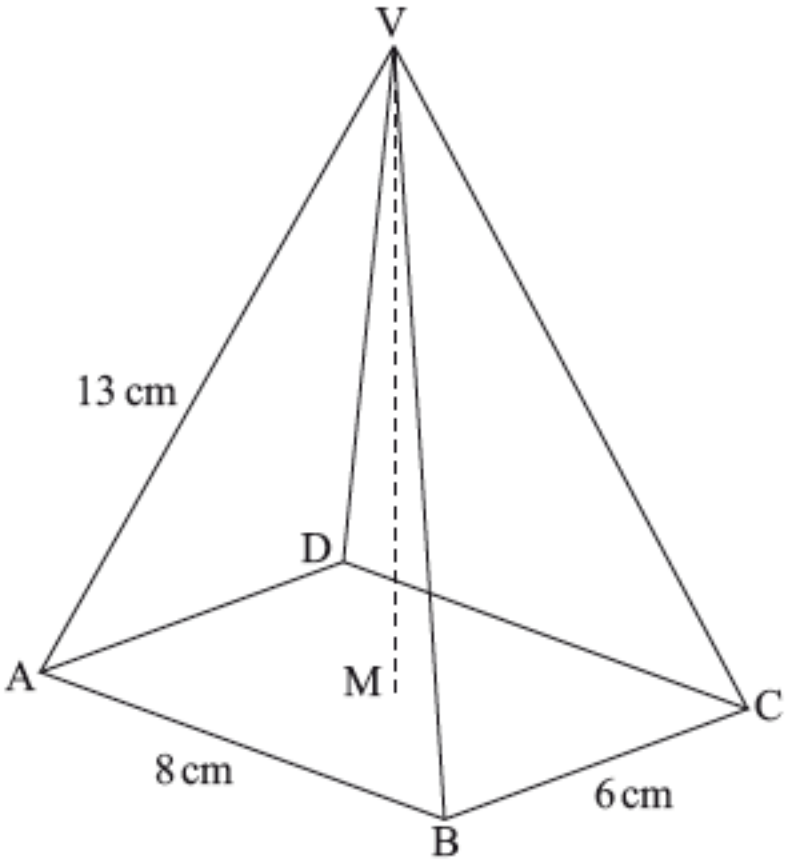
(ii) the *x*-coordinate of point A.

4e. *[2 marks]*

Calculate the value of a.

5a. *[4 marks]*

A right pyramid has apex V and rectangular base ABCD, with $\mathrm{AB} = 8\ \mathrm{cm}$, $\mathrm{BC} = 6\ \mathrm{cm}$ and $\mathrm{VA} = 13\ \mathrm{cm}$. The vertical height of the pyramid is VM.



Calculate VM.

5b. *[2 marks]*

Calculate the volume of the pyramid.

6a. *[1 mark]*

A building company has many rectangular construction sites, of varying widths, along a road.

The area, A, of each site is given by the function

$$A(x) = x(200 - x)$$

where x is the **width** of the site in metres and $20 \leqslant x \leqslant 180$.

Site S has a width of 20 m. Write down the area of S.

6b. *[2 marks]*

Site T has the same area as site S, but a different width. Find the width of T.

6c. *[2 marks]*

When the width of the construction site is b metres, the site has a maximum area.

(i) Write down the value of b.

(ii) Write down the maximum area.

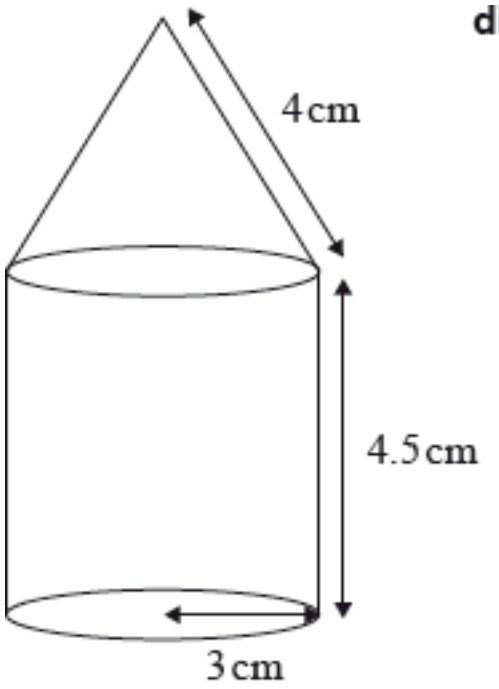
6d. *[1 mark]*

The range of $A(x)$ is $m \leqslant A(x) \leqslant n$.

Hence write down the value of m and of n.

7a. *[6 marks]*

The following diagram shows a perfume bottle made up of a cylinder and a cone.



The radius of both the cylinder and the base of the cone is 3 cm.

The height of the cylinder is 4.5 cm.

The slant height of the cone is 4 cm.

(i) Show that the vertical height of the cone is 2.65 cm correct to three significant figures.

(ii) Calculate the volume of the perfume bottle.

7b. *[2 marks]*

The bottle contains $125\ \text{cm}^3$ of perfume. The bottle is **not** full and all of the perfume is in the cylinder part.

Find the height of the perfume in the bottle.

7c. *[4 marks]*

Temi makes some crafts with perfume bottles, like the one above, once they are empty. Temi wants to know the surface area of one perfume bottle.

Find the **total** surface area of the perfume bottle.

7d. *[4 marks]*

Temi covers the perfume bottles with a paint that costs 3 South African rand (ZAR) per millilitre. One millilitre of this paint covers an area of $7\ \text{cm}^2$.

Calculate the cost, in ZAR, of painting the perfume bottle. **Give your answer correct to two decimal places**.

7e. *[2 marks]*

Temi sells her perfume bottles in a craft fair for 325 ZAR each. Dominique from France buys one and wants to know how much she has spent, in euros (EUR). The exchange rate is 1 EUR = 13.03 ZAR.

Find the price, in EUR, that Dominique paid for the perfume bottle. **Give your answer correct to two decimal places**.

8a. *[1 mark]*

The length of a square garden is $(x + 1)$ m. In one of the corners a square of 1 m length is used only for grass. The rest of the garden is only for planting roses and is shaded in the diagram below.

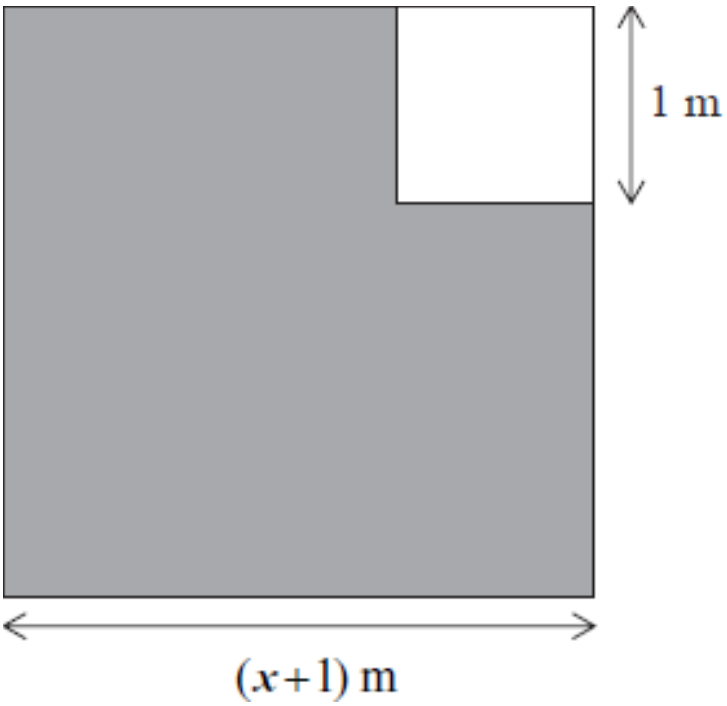


diagram not to scale

The area of the shaded region is A .

Write down an expression for A in terms of x.

8b. *[3 marks]*

Find the value of x given that $A = 109.25$ m.

8c. *[2 marks]*

The owner of the garden puts a fence around the shaded region. Find the length of this fence.

9a. *[2 marks]*

The area of a circle is equal to 8 cm.

Find the radius of the circle.

9b. *[1 mark]*

This circle is the base of a **solid** cylinder of height 25 cm.

Write down the volume of the **solid** cylinder.

9c. *[3 marks]*

This circle is the base of a **solid** cylinder of height 25 cm.

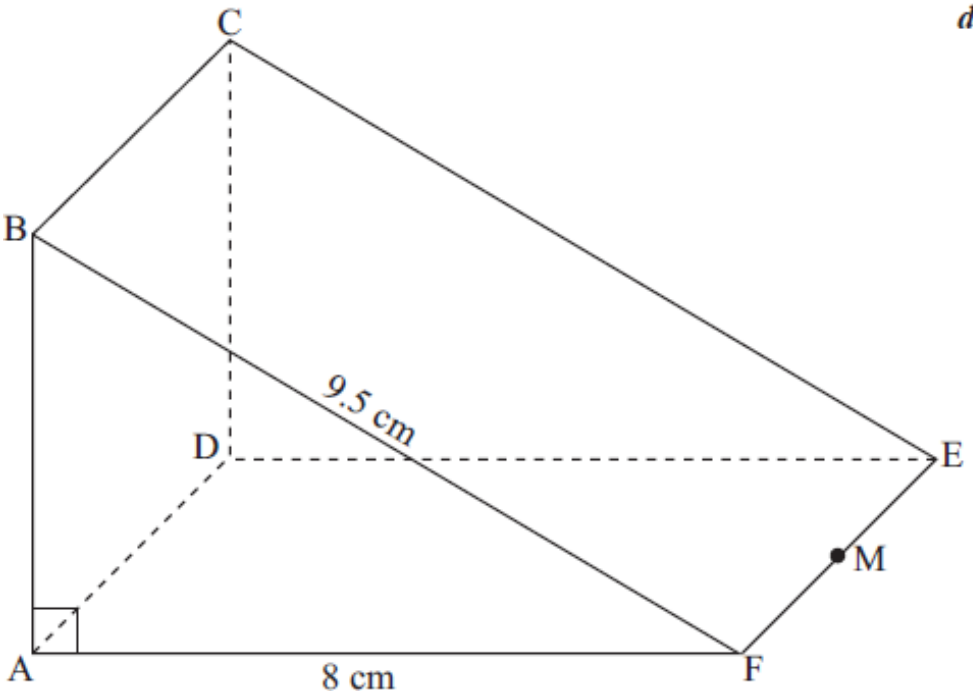
Find the **total** surface area of the **solid** cylinder.

10a. *[2 marks]*

The diagram shows a right triangular prism, ABCDEF, in which the face ABCD is a square.

AF = 8 cm, BF = 9.5 cm, and angle BAF is 90°.

diagram not to scale



Calculate the length of AB .

10b. *[2 marks]*

M is the midpoint of EF .

Calculate the length of BM .

10c. *[2 marks]*

M is the midpoint of EF .

Find the size of the angle between BM and the face ADEF .

11a. *[2 marks]*

A solid metal **cylinder** has a base radius of 4 cm and a height of 8 cm.

Find the area of the base of the cylinder.

11b. *[2 marks]*

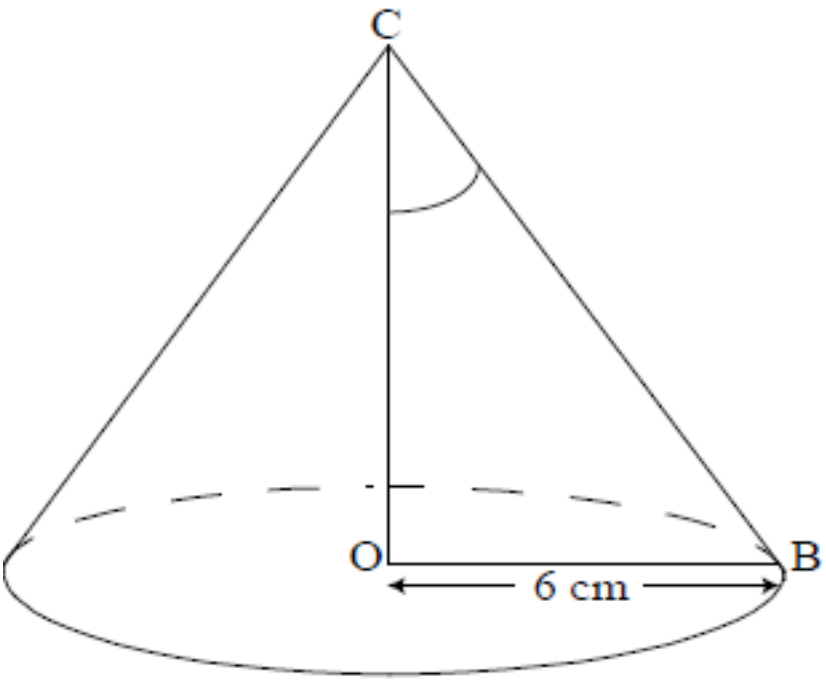
Show that the volume of the metal used in the cylinder is 402 cm, given correct to three significant figures.

11c. *[3 marks]*

Find the total surface area of the cylinder.

11d. *[3 marks]*

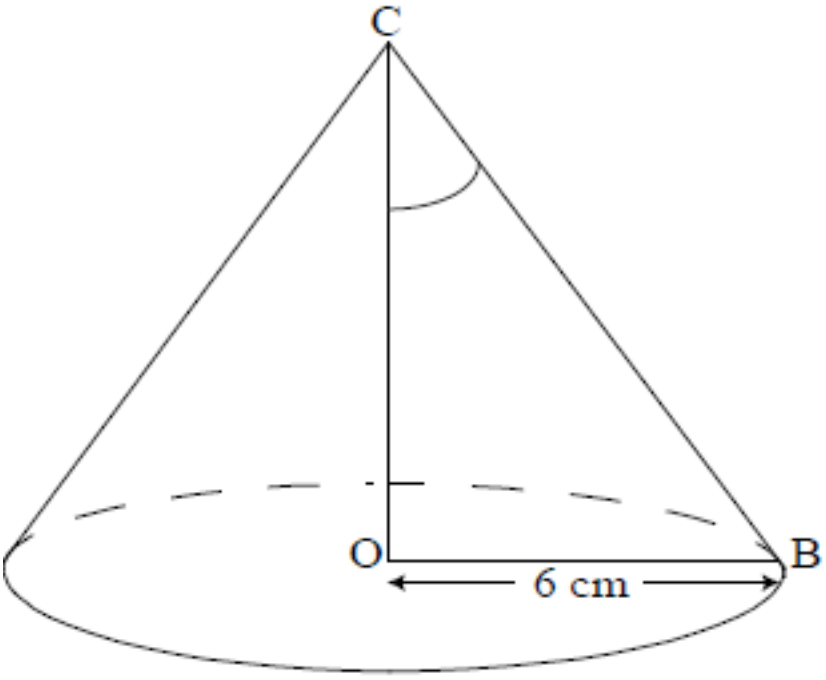
The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm.



Find the height, OC, of the cone.

11e. *[2 marks]*

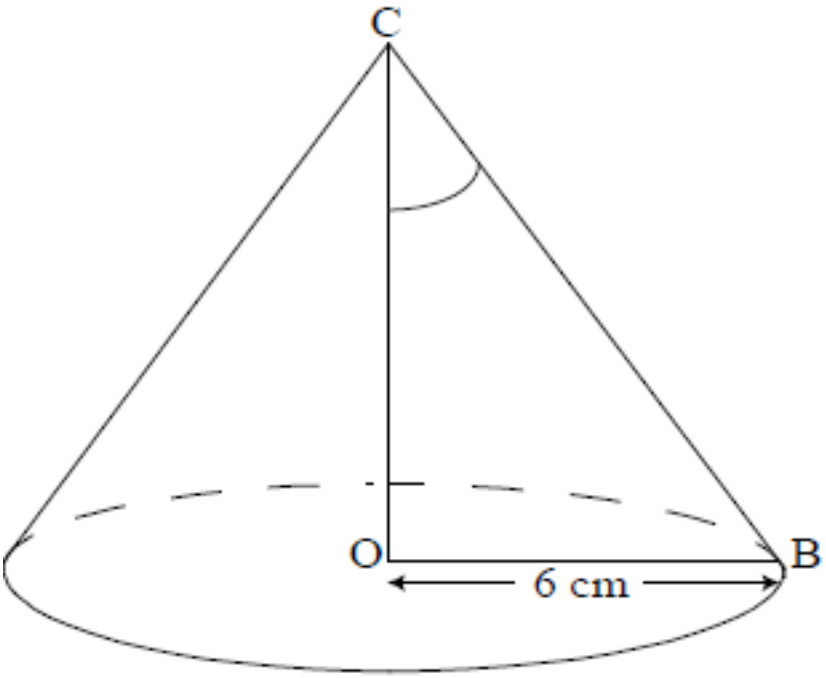
The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm.



Find the size of angle BCO.

11f. *[2 marks]*

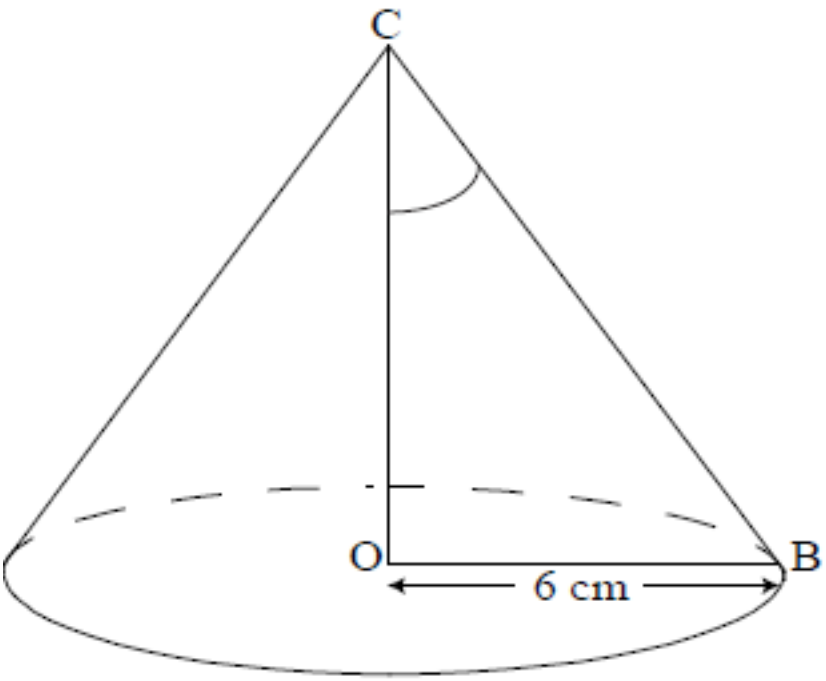
The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm.



Find the slant height, CB.

11g. *[4 marks]*

The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm.

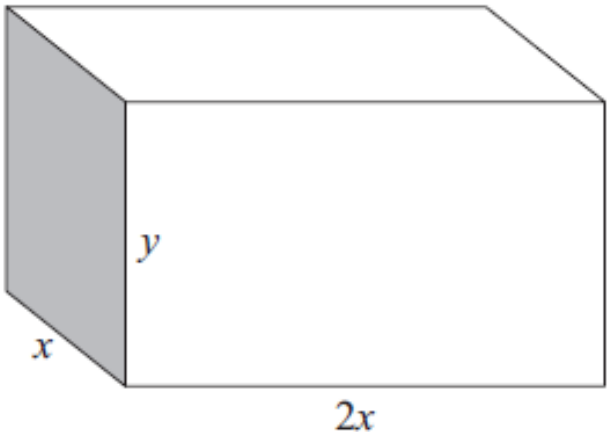


Find the total surface area of the cone.

12a. *[3 marks]*

A shipping container is to be made with six rectangular faces, as shown in the diagram.

diagram not to scale



The dimensions of the container are

length $2x$ width x height y.

All of the measurements are in metres. The total length of all twelve edges is 48 metres.

Show that $y = 12 - 3x$.

12b. *[2 marks]*

Show that the volume V m of the container is given by

$V = 24x - 6x$

12c. *[2 marks]*

Find $\frac{dV}{dx}$.

12d. *[3 marks]*

Find the value of x for which V is a maximum.

12e. *[2 marks]*

Find the maximum volume of the container.

12f. *[3 marks]*

Find the length and height of the container for which the volume is a maximum.

12g. *[4 marks]*

The shipping container is to be painted. One litre of paint covers an area of 15 m . Paint comes in tins containing four litres.

Calculate the number of tins required to paint the shipping container.

13a. *[4 marks]*

The planet Earth takes one year to revolve around the Sun. Assume that a year is 365 days and the path of the Earth around the Sun is the circumference of a circle of radius $150000000\ \mathrm{km}$.

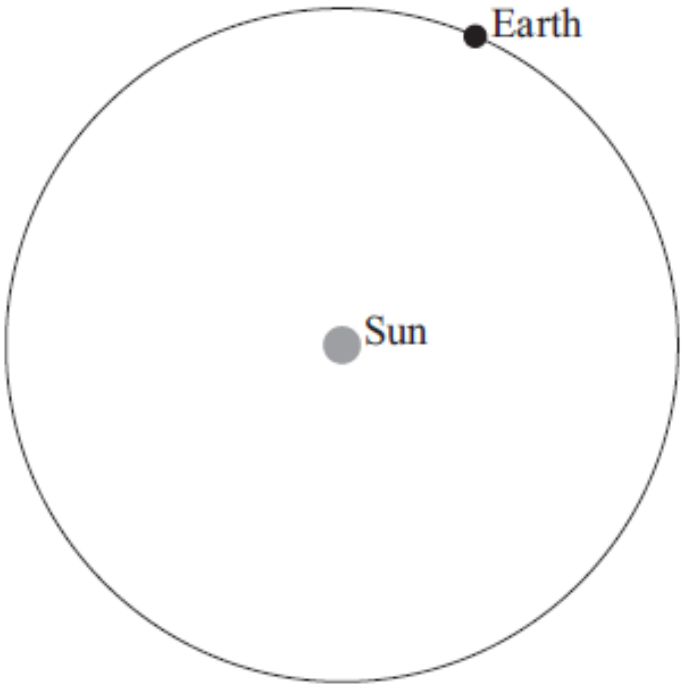


diagram not to scale

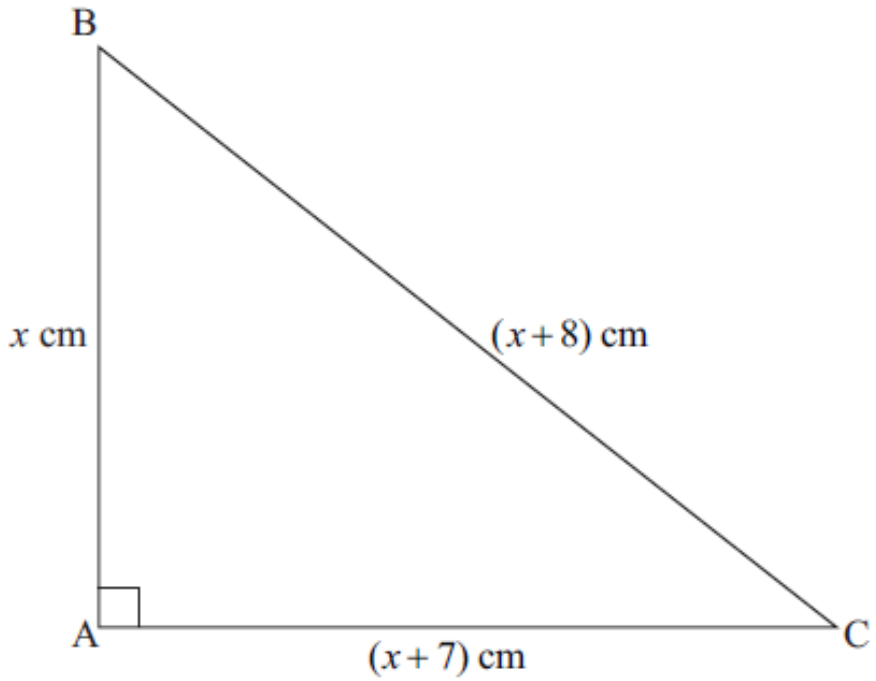
Calculate the distance travelled by the Earth in **one day**.

13b. *[2 marks]*

Give your answer to part (a) in the form $a \times 10^k$ where $1 \leqslant a \leqslant 10$ and $k \in \mathbb{Z}$.

14a. *[3 marks]*

In the diagram, $\mathrm{B\hat{A}C} = 90^\circ$. The length of the three sides are x cm, $(x + 7)$ cm and $(x + 8)$ cm.



Write down and **simplify** a quadratic equation in x which links the three sides of the triangle.

14b. *[2 marks]*

Solve the quadratic equation found in part (a).

14c. *[1 mark]*

Write down the value of the perimeter of the triangle.

15a. *[3 marks]*

A rectangle is 2680 cm long and 1970 cm wide.

Find the perimeter of the rectangle, giving your answer in the form $a \times 10^k$, where $1 \leqslant a \leqslant 10$ and $k \in \mathbb{Z}$.

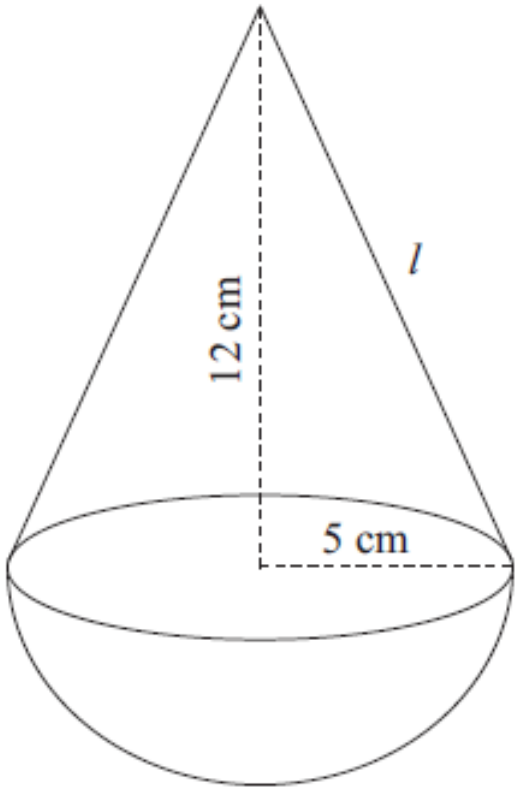
15b. *[3 marks]*

Find the area of the rectangle, giving your answer correct to the nearest thousand square centimetres.

16a. *[2 marks]*

A child's toy consists of a hemisphere with a right circular cone on top. The height of the cone is 12 cm and the radius of its base is 5 cm. The toy is painted red.

diagram not to scale



Calculate the length, l, of the slant height of the cone.

16b. *[4 marks]*

Calculate the area that is painted red.

17a. *[2 marks]*

The diagram shows a pyramid $VABCD$ which has a square base of length $10\ \text{cm}$ and edges of length $13\ \text{cm}$. M is the midpoint of the side BC.

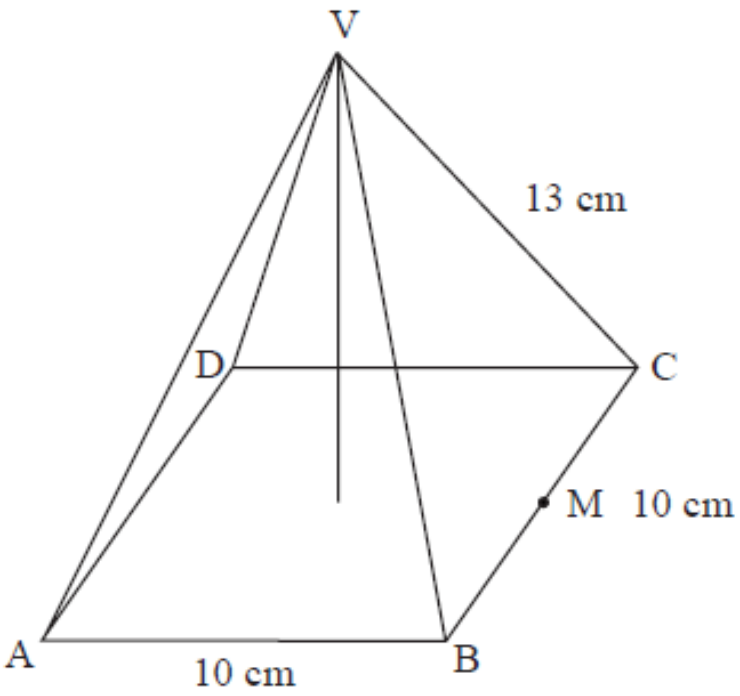


diagram not to scale

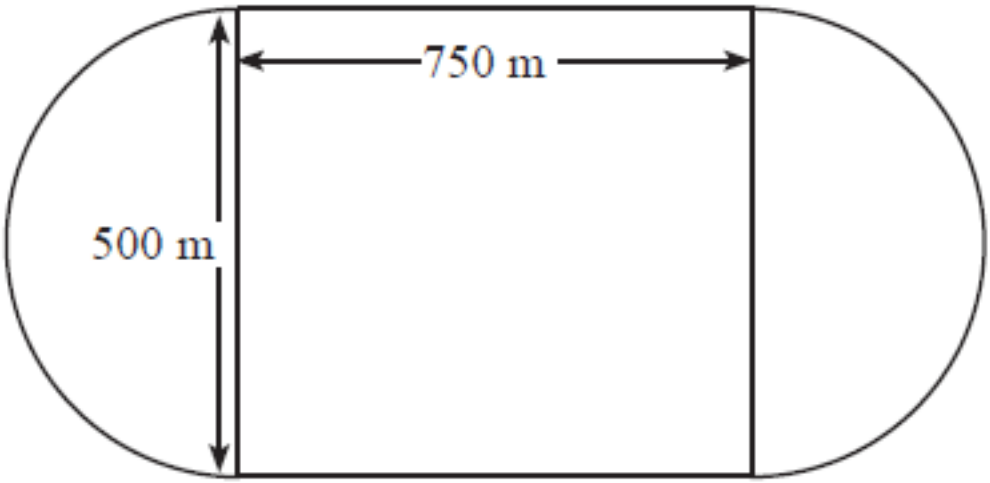
Calculate the length of $\mathbf{VM}$.

17b. *[2 marks]*

Calculate the vertical height of the pyramid.

18a. *[2 marks]*

A race track is made up of a rectangular shape $\mathbf{750m}$ by $\mathbf{500m}$ with semi-circles at each end as shown in the diagram.



Michael drives around the track once at an average speed of $\mathbf{140\ kmh^{-1}}$.

Calculate the distance that Michael travels.

18b. *[4 marks]*

Calculate how long Michael takes in **seconds**.
